

Department of Mathematics and Statistics KFUPM  
STAT 302-02 Quiz#3, Time: 45 mins

Student's Name: \_\_\_\_\_ ID: \_\_\_\_\_

Q.No.1:- Forty workers were randomly divided into two sets of 20 each. Each set spent two weeks in a self-training program that was designed to teach a new production technique. The first set of workers was accompanied by a supervisor whose only job was to check that the workers were all paying attention. The second group was left on its own. The population standard deviations for both the sets are assumed equal. After the program ended, the workers were tested. The following results were as follows:

	<b>Sample Mean</b>	<b>Sample St. Dev.</b>
Supervised group	70.6	8.4
Unsupervised group	77.4	7.4

- a. Derive  $(1 - \alpha)100\%$  confidence interval for the difference between the two population means.

- b. Using the above data, estimate the true difference between the two population means using 95% confidence interval.

Q.No.2:- For air travelers, one of the biggest complaints is of the waiting time between when the airplane taxis away from the terminal until the flight takes off. This waiting time is known to have a uniform distribution over the interval  $[0, \theta]$ . Derive an estimator for  $\theta$  using the method of moments.

Continuous Uniform Distribution:  $f(x) = \frac{1}{b-a}$ ;  $a \leq x \leq b$ ;  $\mu = \frac{b+a}{2}$ ;  $\sigma^2 = \frac{(b-a)^2}{12}$

Q.No.3:- The amount of time required for an oil and filter change on an automobile is normally distributed with a mean of 45 minutes and an unknown standard deviation ( $\sigma$ ). Obtain an estimator for  $2\sigma$  using maximum likelihood estimation.

$$\text{Normal Distribution: } f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2\sigma^2}(x-\mu)^2}; \quad -\infty \leq x \leq \infty$$

$$\bar{x} \pm Z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}} \quad \text{or} \quad \bar{x} \pm Z_{\frac{\alpha}{2}} \frac{s}{\sqrt{n}} \quad \text{or} \quad \bar{x} \pm t_{\frac{\alpha}{2}, n-1} \frac{s}{\sqrt{n}} \quad \text{or} \quad \hat{p} \pm Z_{\frac{\alpha}{2}} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$(\bar{x}_1 - \bar{x}_2) \pm Z_{\frac{\alpha}{2}} \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} \quad \text{and} \quad (\hat{p}_1 - \hat{p}_2) \pm Z_{\frac{\alpha}{2}} \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$$

$$(\bar{x}_1 - \bar{x}_2) \pm t_{\frac{\alpha}{2}, n_1+n_1-2} s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}} \quad \text{where } s_p = \sqrt{\frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}}$$