KING FAHD UNIVERSITY OF PETROLEUM & MINERALS
DEPARTMENT OF MATHEMATICS & STATISTICS
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STAT 302: Statistical Inference

Semester 151, Final Exam Objective

Sunday December 20, 2015, 7:00 − 8:00 pm

Name: ID #:

1. Let $Y_1, Y_2, \dots \dots, Y_n$ be a random sample from uniform distribution over the interval $[0,1]$. The probability density function of the highest order statistic $Y_{(n)}$ is $g_n(y) = ny^{(n-1)}$ where

(a) $1 < y_{(n)} < \infty$ (b) $0 < y_{(n)} < \infty$

(c) $0 < y_{(n)} < 1$ (d) $-\infty < y_{(n)} < \infty$

(e) None of above (f) all of (a), (b), (c) and (d)

2. Let Y be a continuous random variable with density function $f(y) = 6y(1-y); 0 \leq y \leq 1$. If a random sample of size $n = 2$ is taken, the probability that the maximum value of the sample is less than 1 is equal to

(a) 0 (b) 0.6286

(c) 0.9998 (d) 0.5

(e) 0.1317 (f) None of above

3. Let $Y_1, Y_2, \dots \dots, Y_{15}$ be a random sample from $N(\mu, \sigma^2)$. The probability that $\sum_{i=1}^{15} \left(\frac{Y_i - \mu}{\sigma} \right)^2$ is greater then 8.55 is approximately

(a) 0 (b) 0.3

(c) 0.9 (d) 0.5

(e) 0.7 (f) None of above

4. Let $Y_1, Y_2, \dots \dots, Y_{15}$ be a random sample from $N(\mu, \sigma^2)$. The quantity $\frac{\sum_{i=1}^{15} (Y_i - \overline{Y})^2}{\sigma^2}$ follows

(a) $\chi^2_{(15)}$ (b) $\chi^2_{(1)}$

(c) $\chi^2_{(14)}$ (d) $\chi^2_{(13)}$

(e) $\chi^2_{(16)}$ (f) None of above

5. An estimator $\hat{\theta}$ of population parameter θ converges in probability to θ as n tends to infinity is said to be

(a) Unbiased (b) Consistent

(c) Efficient (d) Sufficient

(e) Minimum Variance Unbiased Estimator (MVUE) (f) None of above

6. Bias of an estimator can be

(a) Positive
(b) Negative
(c) either (a) or (b)
(d) always zero
(e) None of above

6. Cramer-Rao inequality with regard to the variance of an estimator provides

(a) Upper bound on the variance
(b) Lower bound on the variance
(c) Asymptotic variance of an estimator
(d) None of above

7. Mean Squared Error (MSE) of an estimator $\hat{\theta}$ is expressed as

(a) $\text{Bias}(\hat{\theta}) + \text{Var}(\hat{\theta})$
(b) $[\text{Bias}(\hat{\theta}) + \text{Var}(\hat{\theta})]^2$
(c) $[\text{Bias}(\hat{\theta})]^2 + \text{Var}(\hat{\theta})$
(d) $\text{Bias}(\hat{\theta}) + [\text{Var}(\hat{\theta})]^2$
(e) $[\text{Bias}(\hat{\theta})]^2 + [\text{Var}(\hat{\theta})]^2$
(f) None of above

8. Let $Y_1, Y_2, \dots \dots, Y_n$ be a random sample from a population $f(y) = \frac{1}{\theta\sqrt{2\pi}} e^{-\frac{y^2}{2\theta^2}}; -\infty < y < \infty$. Then the Maximum Likelihood Estimator (MLE) for θ is

(a) $\frac{\sum Y_i}{n}$
(b) $\frac{\sum Y_i^2}{n}$
(c) $\frac{\sqrt{\sum Y_i}}{n}$
(d) $\frac{\left(\sum Y_i\right)^2}{n}$
(e) $\sqrt{\frac{\sum Y_i^2}{n}}$
(f) None of above

9. Formula for 95% confidence limits for the variance of population $N(\mu, \sigma^2)$, when μ is unknown, is:

(a) $P\left[\chi^2_{1-\alpha/2} \leq \frac{ns^2}{\sigma^2} \leq \chi^2_{\alpha/2}\right] = 1 - \alpha$
(b)) $P\left[\frac{ns^2}{\chi^2_{\alpha/2}} \leq \sigma^2 \leq \frac{ns^2}{\chi^2_{1-\alpha/2}}\right] = 1 - \alpha$
(c) $P\left[\frac{ns^2}{\chi^2_{1-\alpha/2}} \geq \sigma^2 \geq \frac{ns^2}{\chi^2_{\alpha/2}}\right] = \alpha$
(d) none of above
(e) both (a) and (b)
(f) all of (a), (b) and (c)

10. Let X represents a random variable following $N(\mu, \sigma^2 = 4)$. The hypotheses are $H_0: \mu = 2$ against $H_1: \mu = 1/2$. A sample of size 25 is selected randomly and H_0 is rejected if sample mean is less than 1.
Then the size of type I error is equal to ____________ and the power of the test is equal to ___________.

11. A sample of 36 measurements shows a standard deviation of 0.07. We have to test the hypothesis that the true standard deviation is 0.05 against that it is not. Using 5% level of significance, the test reveals that

(a) H_0 is not rejected
(b) H_0 is rejected
(c) Not possible to test
(d) None of above

12. Let $Y_1, Y_2, \dots \dots, Y_{45}$ be a random sample from exponential distribution with $\lambda = 0.5$. The probability that the sample mean will be between 1.8 and 2.1 is equal to

(a) 0.13 (b) 0.61
(c) 0.94 (d) 0.38
(e) 0.77 (f) None of above

13. Let $Y_1, Y_2, \dots \dots, Y_n$ be a random sample from $N(\mu, \sigma^2)$. If we define $S^2 = \frac{1}{n}\sum_{i=1}^{n}(Y_i - \bar{Y})^2$, then an unbiased estimator for σ^2 is

(a) $\frac{1}{(n-1)}S^2$ (b) $\frac{1}{n}S^2$
(c) $\frac{(n-1)}{n}S^2$ (d) $\frac{n}{(n-1)}S^2$
(e) nS^2 (f) None of above

14. Let $Y_1, Y_2, \dots \dots, Y_n$ be a random sample from $N(\mu, \sigma^2)$. Then the efficiency of sample median relative to sample mean ($eff(\tilde{X}, \bar{X}) = \frac{Var(\bar{X})}{Var(\tilde{X})}$) is

(a) Less than one (b) Greater than one
(c) Equal to one (d) Does not exist
(e) None of above

15. Let $Y_1, Y_2, \dots \dots, Y_n$ be a random sample from a Bernoulli population with density function $f(y) = p^y(1-p)^{1-y}$; $y = 0,1$. A sufficient statistic for p is

(a) $\frac{y_{(1)} + y_{(n)}}{2}$ (b) $\prod_{i=1}^{n} Y_i$
(c) $\min(y_1, y_2, \dots, y_n)$ (d) $\max(y_1, y_2, \dots, y_n)$
(e) $\sum_{i=1}^{n} Y_i$ (f) None of above

16. If U is a sufficient statistic that best summarizes the data and some function of U (say $h(U)$) can be found such that $h(U)$ is a/an ____________________ estimator of unknown parameter θ, then $h(U)$ is Minimum Variance Unbiased Estimator (MVUE) for θ.

17. A type I error is made if H_0 is rejected when H_0 is ____________. The probability of a type I error is denoted by ___________.

18. Let $Y_1, Y_2, \dots \dots, Y_{15}$ be a random sample from $N(\mu, \sigma^2)$ with unknown σ^2. For testing $H_0: \mu = \mu_0$ against $H_1: \mu < \mu_0$, we will reject H_0 if ____________________.

19. In sign test, the test statistic ($K =$ No. of positive signs) follows a binomial distribution with parameters n and $p =$__________________.

20. Prior distributions that result in posterior distributions that are of the same functional form as the prior but with altered parameter values are called ____________________________________.