KING FAHD UNIVERSITY OF PETROLEUM & MINERALS
DEPARTMENT OF MATHEMATICS & STATISTICS
DHAHRAN, SAUDI ARABIA

STAT 302: Statistical Inference
Semester 151
Second Major Exam
Saturday October 31, 2015
5:30 – 7:30 pm

Name: ID #:

Question No	Full Marks	Marks Obtained
1	**10**	
2	**07**	
3	**11**	
4	**6**	
Total	**34**	

Q.No.1:- (2×5 = 10 points) Let $Y_1, Y_2, \dots\dots, Y_5$ be a random sample of size 8 from a normal population with mean μ and variance $\sigma^2 = 1$. Let Y_9 be another independent observation from the same population. What is the distribution (along with the degree of freedom) of:

a) $W_1 = \sum_{i=1}^{8} Y_i^2$? Why?

Reason:_____

_____

b) $W_2 = \sum_{i=1}^{8} (Y_i - \bar{Y})^2$? Why?

Reason:_____

_____

c) $W_3 = \sum_{i=1}^{8} (Y_i - \bar{Y})^2 + Y_9^2$? Why?

Reason:_____

_____

d) $W_4 = \frac{\sqrt{8}(\bar{Y}-\mu)}{\sqrt{\sum_{i=1}^{8}(Y_i-\bar{Y})^2 / 7}}$? Why?

Reason:_____

_____

e) $W_5 = \frac{Y_9^2}{\sum_{i=1}^{8}(Y_i-\bar{Y})^2 / 7}$? Why?

Reason:_____

_____

Q.No.2:- (3+4 = 7 points) An important aspect of a federal economic plan was that consumers would save a substantial portion of the money from an income tax reduction. Suppose that the tax saved by customers had mean 2600 SR and standard deviation 120 SR. If a random sample of 35 customers is selected,

a) What will be the distribution of sample mean $\bar{Y}$? Justify your answer? Also specify all the parameters of the distribution.

b) What is the probability that the sample mean will lie between 2510 and 2630?

Q.No.3:- (4+2+5 = 11 points) Age of the car is an important factor for the car insurance companies. Suppose that the age of car is uniformly distributed over the interval $(\theta - 1, \theta + 2)$, where θ is the unknown parameter. Suppose that $Y_1, Y_2, \ldots \ldots, Y_9$ denote a random sample of car ages.

a) Show that $\bar{Y}$ is a biased estimator of θ. Also compute the Bias and Mean Square Error of $\bar{Y}$.

b) Find a function of $\bar{Y}$ that is an unbiased estimator of θ.

c) Suppose $\hat{\theta}_1 = \bar{Y} - 0.5$ and $\hat{\theta}_2 = \frac{Y_1 - Y_2 - 1}{2} - Y_9$ are two unbiased estimators of θ. Find the efficiency of $\hat{\theta}_1$ relative to $\hat{\theta}_2$.

Q.No.4:- (3+3 = 6 points) If $Y_1, Y_2, \ldots \ldots, Y_n$ denote a random sample from a geometric distribution with parameter p, show that:

a) $\bar{Y}$ is consistent for p.

b) $\bar{Y}$ is sufficient for p.