Math 514-151 Homework 3 (Time: 12 days)

Name:.....ID#:.....

Q.1: Find the Fourier transform of $f(x) = \left(1 - \frac{|x|}{a}\right) H\left(1 - \frac{|x|}{a}\right)$, where $H(x)$ is the heaviside unit step function.

Q.2: Find the Fourier transform of (a) $f(x) = \frac{1}{x^2+4}$ and (b) $f(x) = \frac{x}{x^2+4}$.

Q.3: Solve the integral equation $\int_{-\infty}^{\infty} f(x-t)e^{-at}dt = \frac{1}{x^2+b^2} - f(x)$ using Fourier transform to find $f(x)$.

Q.4: Using appropriate Fourier transform, solve the initial value problem

$u_t = ku_{xx}$, $-\infty < x < \infty$, $t > 0$ with the initial condition $u(x,0) = f(x)$, $-\infty < x < \infty$.

Q.5: Using appropriate Fourier transform, solve the initial-boundary value problem

$u_t = ku_{xx}$, $0 < x < \infty$, $t > 0$ with the initial condition $u(x,0) = 0$, $0 < x < \infty$ and the boundary conditions $u(0,t) = T_o$, $t \geq 0$ and $u(x,t) \to \infty$ as $x \to \infty$.

Note: $\int_0^{\infty} \frac{\sin \alpha x}{\alpha} d\alpha = \frac{\pi}{2}$ and $\int_0^{\infty} e^{-\alpha^2 a^2} \frac{\sin \alpha x}{\alpha} d\alpha = \frac{\pi}{2} erf(\frac{x}{2a})$.

Q.6: Using appropriate Fourier transform, solve the initial-boundary value problem

$u_{tt} = c^2 u_{xx}$, $0 < x < \infty$, $t > 0$ with the initial conditions $u(x,0) = 0$, $u_t(x,0) = 0$, for $0 < x < \infty$ and boundary condition $u(0,t) = f(t)$ for $t > 0$.