Math 514-151   Final Exam   (Time: 150 minutes)

Name:...ID#:...............................

Q:1 (25 points) Use Laplace transform to solve $u_{xx} = u_{tt} - xe^{-t}$, $0 < x < \infty$, $t > 0$

under the following conditions

$$u(0,t) = \cos t, \text{ and } \lim_{x \to \infty} |u(x,t)| \sim x^n \text{ for some } n \text{ and } t > 0$$

$$u(x,0) = 1 \text{ and } u_t(x,0) = 0, \text{ for } 0 < x < \infty$$

Q:2 (25 points) Use Hankel transform to solve the wave equation

$$\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} = \frac{1}{C^2}\frac{\partial^2 u}{\partial t^2}, \quad 0 < r < \infty, \quad t > 0,$$

subject to the initial conditions

$$\begin{aligned} u(r,0) &= f(r) \\ u_t(r,0) &= g(r). \end{aligned}$$

Q.3: (25 points) Show that $\mathcal{H}\{f'(r)\} = \dfrac{\alpha}{2n}\left[(n-1)\tilde{f}_{n+1}(\alpha) - (n+1)\tilde{f}_{n-1}(\alpha)\right]$, $n \geq 1$

Q.4: (25 points) Solve using Mellin transform

$$x^2\frac{\partial^2 u}{\partial x^2} + x\frac{\partial u}{\partial x} + \frac{\partial^2 u}{\partial y^2} = 0, \quad 0 \leq x < \infty, \quad 0 < y < 1,$$

subject to the conditions

$$\begin{aligned} u(x,0) &= 0 \\ u(x,1) &= \begin{cases} A, & 0 \leq x \leq 1 \\ 0, & x > 1 \end{cases} \end{aligned}$$

Q.5: (20 points) Find and sketch image of the vertical strip $-\dfrac{\pi}{2} \leq x \leq \dfrac{\pi}{2}$ under the

mapping $w = f(z) = \sin z$. Check if f is conformal or not.

Q.6: (20 points) Find a harmonic function $\Phi(x,y)$ in the upper half of the z-plan which satisfy

$$\Phi(x,0) = G(x) = \begin{cases} A, & x > 0 \\ 0, & x < 0 \end{cases}$$