Math 514-151 Exam 2 (Time: 120 minutes)

Name:.....ID#:.....

Q:1 (30 points) Use appropriate Fourier transform to solve the Laplace equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0, \quad 0 < x, y < \infty$$

subject to the boundary conditions

$$\begin{aligned} u(0, y) &= a, \quad u(x, 0) = 0, \\ \nabla u \to & \quad 0 \quad as \;\; r \to \infty. \end{aligned}$$

where a is a constants.

Q:2 (30 points) Use appropriate Fourier transform to solve the heat equation

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}, \quad -\infty < x < \infty$$

subject to the initial and boundary conditions

$$\begin{aligned} u(x, 0) &= U_0, \; constant \\ u(x, t) \to & \quad 0 \quad as \;\; |x| \to \infty \\ u_x(x, t) \to & \quad 0 \quad as \;\; |x| \to \infty. \end{aligned}$$

Q.3: (20 points) Use Fourier transform to solve the integral equation

$$\int_{-\infty}^{\infty} \frac{f(t)}{(x-t)^2 + t^2} dt = \frac{1}{x^2 + b^2}$$

Q.4: (20 points) Show that the Fourier transform of $f(x) = xe^{-a|x|}$ is $F(\alpha) = -\dfrac{4a\alpha i}{(\alpha^2 + a^2)^2}$