Math 514-151   Exam 1   (Time: 120 minutes)

Name:..ID#:..............................

Q.1: (20 points) Evaluate the contour integral $\oint_C \dfrac{\sin \pi z^2 + \cos \pi z^2}{(z-2)(z-1)} \, dz$ where C is the circle $|z| = 3$.

Q.2: (20 points) Evaluate $\displaystyle\int_{-\infty}^{\infty} \dfrac{5x^2}{(x^2-1)^2(x^2+2x+2)} \, dx$

Q.3: (20 points) Find inverse Laplace transform of $F(s) = \dfrac{1}{s \sinh(as)}$ using contour integration.

(Note: Zeros of $\sinh x$ are $x = \pm n\pi i$, $n = 0, 1, 2, \cdots$)

Q.4: (15 points) Find $f(t)$ from the integral equation $f(t) = 8t^2 - 3 \int_0^t f(\tau) \sin(t - \tau) \, d\tau$

Q:5 (25 points) Use Laplace transform to solve the inhomogeneous wave equation

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{c^2} \frac{\partial^2 u}{\partial^2 t} - k \sin\left(\frac{\pi x}{a}\right), \quad 0 < x < a, \ t > 0,$$

subject to the boundary and initial conditions

$$
\begin{aligned}
u(0, t) &= 0, & u(a, t) = 0, \quad t > 0, \\
u(x, 0) &= 0, & u_t(x, 0) = 0, \quad 0 < x < a.
\end{aligned}
$$

where c, k and a are constants.