Project: 04

1) Consider the problem

$$\text{minimize} \quad x_2 - (x_1 - 2)^3 + 3$$
$$\text{Subject to} \quad x_2 \geq 1.$$

(i) Write down the KKT conditions and find all points that satisfy the condition. Check whether or not each point is regular.

(ii) Determine whether or not the point(s) in part (i) satisfy the second order necessary conditions.

(iii) Determine whether or not the point(s) in part (ii) satisfy the second order sufficient conditions.

2) Solve the Following optimization problem using the second order sufficient conditions:

$$\text{Minimize} \quad x_1^2 + x_2^2$$
$$\text{Subject to} \quad x_1^2 - x_2 - 4 \leq 0$$
$$x_2 - x_1 - 2 \leq 0.$$

3) Consider the Following problem:

$$\text{Minimize} \quad (x_1 - \tfrac{9}{4})^2 + (x_2 - 2)^2$$
$$\text{Subject to} \quad x_2 - x_1^2 \geq 0$$
$$x_2 + x_1 \leq 6$$
$$x_1, x_2 \geq 0.$$

(i) Write the KKT optimality conditions and verify that these conditions hold true at the point $\bar{x} = (\frac{3}{2}, \frac{9}{4})$

(ii) Interpret the KKT conditions at $\bar{x}$ graphically.

(iii) Show that $\bar{x}$ is indeed the unique global optimal Solution.