

## HW # 1

① Consider the graphical representation of the following linear programs

Maximize (or minimize)  $Z = 3x_1 + 2x_2$

Subject to  $x_1 + 2x_2 \geq 4$ ,  $x_1 \leq 3$ ,  $x_2 \leq 2$ ,  $x_1 + x_2 \geq 5$ ,  $x_1, x_2 \geq 0$ .

(a) In each of the following cases indicate if the feasible region has one point, infinite number of points, or no points.

(i) The constraints are as given above.

(ii) The constraint  $x_1 + x_2 \geq 5$  is changed to  $x_1 + x_2 \geq 3$ .

(iii) " " " " " " " "  $x_1 + x_2 \geq 7$ .

(b) For each case in (a), ~~in which~~ determine the number of feasible extreme points, if any.

(c) For the cases in (a) in which a feasible solution exists determine the maximum and minimum values of  $Z$  and their associated extreme points.

(2) Solve graphically

(i) Max  $Z = x_1 + x_2$

subject to  $x_1 + 5x_2 \leq 5$ ,  $2x_1 + x_2 \leq 4$ ,  $x_1, x_2 \geq 0$

Ans:  
 $(\frac{5}{3}, \frac{2}{3})$   
 $Z = \frac{7}{3}$

(ii) Max  $Z = 2x_1 + 3x_2$

s.t.  $x_1 + x_2 \leq 4$ ,  $2x_1 + x_2 \geq 2$ ,  $x_1, x_2 \geq 0$

Ans:  $(0, 4)$

(iii) Min  $Z = 4x_1 + 2x_2$

s.t.  $x_1 + x_2 \leq 4$

$x_1 + 2x_2 \geq 2$

$2x_1 + x_2 \geq 2$

$x_1 \leq 3$

$x_2 \leq 3$

$x_1, x_2 \geq 0$

Ans: All the points on the line segment joining  $(0, 2)$  and  $(\frac{2}{3}, \frac{2}{3})$

(3) Max  $Z = x_1 + x_2$

s.t.  $3x_1 - 4x_2 \geq -3$

$-x_1 - x_2 \leq 0$

$x_1, x_2 \geq 0$

Ans: unbounded solution.