Final Version

King Fahd University of Petroleum and Minerals

Department of Mathematics and Statistics

Math 480
Final Exam – 2015–2016 (151)
Allowed Time: 180 minutes

Name: ____

ID #: __________________________

Section #: _____________ Serial Number: ______________

Instructions:

1. Write clearly and legibly. You may lose points for messy work.
2. **Show all your work**. No points for answers without justification.
3. **Calculators and Mobiles are not allowed.**

Written Problems

Question #	Grade	Maximum Points
1		8
2		7
3		10
4		10
5		11
6		12
7		11
8		11
Total:		80

Q:1 (8 points) Convert the following fractional problem into general linear programming problem and then solve by **Two Phase Method**

$$Maximize \;\; z = \frac{2x_1 + 3x_2}{x_1 + x_2 + 1}$$

subject to

$$\begin{aligned} 2x_1 + x_2 \; &\leq 3 \\ x_1 + 2x_2 &\leq 3 \\ x_1, x_2 &\geq 0. \end{aligned}$$

Q:2(7 points) Let $f \in c^1$. Prove that f is convex function over a convex set S if and only if

$$f(x) \geq f(\bar{x}) + \nabla f(\bar{x})(x - \bar{x}) \text{ for all } x, \bar{x} \in S.$$

Q:3(10 points) Find the minimizer of

$$f(x_1, x_2) = \frac{1}{2}x^T \begin{pmatrix} 4 & 2 \\ 2 & 2 \end{pmatrix} x - x^T \begin{pmatrix} -1 \\ 1 \end{pmatrix}, \; x \in R^2,$$

using the **Conjugate Direction Method** to perform 02 iterations with initial point $(0, 0)$ and congugate directions$[1, 0]$ and $[\frac{-3}{8}, \frac{3}{4}]$.

Q:4(3 + 4 + 3 points) (a) Show directly from the definition whether $f(x_1, x_2) = x_1^2 + x_2^2 - x_1x_2$ is convex function over $S = \{x : x_1 > 0, x_2 > 0\}$ or not ?
(b) Define feasible direction. State the first order necessary conditions of a real valued function f defined on a subset of R^n.
(c) State the second order sufficient conditions for *minimize* $f(x)$; subject to $h(x) = 0$, where $f : R^n \to R$ and $h : R^n \to R^m$ are twice continously differentiable functions.

Q:5(11 points) State the second order sufficient conditions for *minimize* $f(x)$; subject to $h(x) = 0, g(x) \leq 0$) where $f : R^n \to R$, $h : R^n \to R^m$ and $g : R^n \to R^p$ are twice continously differentiable functions. Use these conditions to verify that $(\frac{1}{2}, \frac{3}{2})$ is a strict minimizer of

$$Minimize\ f(x_1, x_2) = x_2 - 2 + (x_1 - 1)^2$$

subject to

$$x_2 - x_1 = 1$$
$$x_1 + x_2 \leq 2$$

for $\lambda = -1$ and $\mu = 0$.

Q:6(6+ 6 points) Consider the following nonlinear program:

Minimize $(x_1 - 2)^2 + (x_2 - 1)^2$,

where x_1 and x_2 are real decesion variables.

(a) Starting from the point $(0, 1)$ perform next iteration of the **Newton's Method**.

(b) Adding the constraints: $-x_1^2 + x_2 \geq 0; -x_1 - x_2 + 2 \geq 0; x_1 \geq 0; x_2 \geq 0$, solve the corresponding constrained problem by KKT conditions.

Q:7 (11 points) Apply **Wolfe's Method** for solving the quadratic programming problem:

$Maximize \;\; z = 4x_1 + 6x_2 - 2x_1^2 - 2x_1x_2 - 2x_2^2$

subject to

$$x_1 + 2x_2 \leq 2$$
$$x_1, x_2 \geq 0.$$

Q:8 (11 points) Starting with $x_0 = (0, 1)$ find the next iteration in approximating the optimal solution of the following problem:

$$\begin{aligned} &Minimize \quad = 2x_1^2 \ + \ 2x_2^2 \ - \ 2x_1x_2 \ - \ 4x_1 \ - \ 6x_2 \\ &\text{subject to} \\ &\qquad x_1 + x_2 \ \leq 2 \\ &\qquad x_1 + 5x_2 \leq 5 \\ &\qquad x_1, x_2 \geq 0 \end{aligned}$$

using **Gradient Projection Method**.