Department of Mathematics and Statistics, KFUPM
Math-480 Semester-151 Major I

Maximum Marks: 60 Time Allowed: 120 minutes

(1)(12 *points*) Old hens can be bought at SAR 2 each and young ones at SAR 5 each. The old hens lay 3 eggs per week and young ones lay 5 eggs per week, each eggs being worth 30 halala. A hen costs SAR 1 per week to feed. I have only SAR 80 to spend for hens, how many of each kind should I buy to give a profit of more than SAR 6 per week, assuming that I cannot house more than 20 hours.

(2*a*)(08 *points*) Define basic feasible solutions of a linear programming problem. Find all basic feasible solutions for the system of simultaneous equations.

$$x_1 - x_2 + 2x_3 + 2x_4 = 4$$
$$3x_1 + 4x_3 + 6x_4 = 1.$$

(2*b*)(08 *points*) Show that feasible region of every linear programming problem is convex set.

(3)(12 *points*) Solve by Simplex Method

$$Maximize \;\; z = -x_1 + 3x_2 - 2x_3$$

subject to

$$\begin{aligned} 3x_1 - x_2 + 2x_3 &\leq 7 \\ -2x_1 + 4x_2 &\leq 12 \\ -4x_1 + 3x_2 + 8x_3 &\leq 10 \\ x_1, x_2, x_3 &\geq 0. \end{aligned}$$

(4)(08 *points*) Write the dual of the following linear programming problem

$$Maximize \;\; z = 3x_1 + x_2 + 2x_3 - x_4$$

subject to

$$\begin{aligned} 2x_1 - x_2 + 3x_3 + x_4 &= 1 \\ x_1 + x_2 - x_3 + x_4 &= 3 \\ x_1, x_2, x_3 &\geq 0. \text{ , } x_4 \text{ is unrestricted.} \end{aligned}$$

(5)(12 *points*) What are artificial variables? Why do we need them ? Use **two-phase simplex method** to solve:

$$Maximize \;\; z = -x_1 - x_2$$

subject to

$$\begin{aligned} 2x_1 + x_2 &\geq 4 \\ x_1 + 7x_2 &\geq 7 \\ x_1, x_2 &\geq 0. \end{aligned}$$