King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics
Math 470 Quiz 4

Name : ..... **ID #**.....

Question: Let $D(x,\epsilon)$ be the disk in R^2 centered at x and with radius ϵ. Let n denote the exterior unit normal to $\partial D(x,\epsilon) = C(x,\epsilon)$.

1. Prove that $\partial_n \ln(\frac{1}{|y-x|}) = -\frac{1}{\epsilon}$ for all $y \in C$.

2. Prove for a harmonic function u that

$$\frac{1}{2\pi}\int_{C(x,\epsilon)} \left(\partial_n u(y) \ln(\frac{1}{|y-x|}) - u(y)\partial_n \ln(\frac{1}{|y-x|})\right) ds = \frac{1}{2\pi\epsilon}\int_{C(x,\epsilon)} u(y) ds.$$