King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics

MATH 470 - Final Exam - Term 151

Duration: 140 minutes

Name:_____ ID Number:_____

Question Number	Points	Maximum Points
1		5
2		5
3		5
4		5
5		5
6		5
7		5
8		5
Total		40

1. Give an example of a Cauchy data for the PDE

$$xu_x + 2x^2\, u_y - u = x^2\, e^x,$$

making it to have no solution.

2. Use characteristics to obtain an implicit solution for the Cauchy problem

$$\partial_t T = -T\partial_x T,$$

$$T(x,0) = x^2.$$

3. Use characteristics to solve

$$w_{yy} = w_{xx}, \quad x \in \mathbb{R}, \quad y > 0$$

$$w(x,0) = f(x), \quad w_y(x,0) = g(x), \quad x \in \mathbb{R}$$

4. Show that

$$u(x,y) = \frac{y}{\pi}\int_{-\infty}^{\infty} \frac{f(s)}{y^2+(s-x)^2}\,ds$$

satisfies

$$\nabla^2 u = 0, \quad x \in \mathbb{R}, \quad y > 0.$$
$$u(x,0) = f(x), \quad x \in \mathbb{R}.$$

5. a) Use question 4 to solve the Neumann problem:

$$\nabla^2 u = 0, \quad x \in \mathbb{R}, \quad y > 0$$

$$\partial_y u(x, 0) = g(x), \quad x \in \mathbb{R}$$

b) Is the solution in part a) unique?

6. Let D be the unit disk in $\mathbb{R}^2$, and let $u = u(x, y)$ be a solution of

$$-\triangle u + u = 1 \quad \text{in} \quad D$$

$$\partial_n u = \sin(xy) \quad \text{on} \quad \partial D.$$

Is the solution unique?

7. Let D be the unit disk in $\mathbb{R}^2$, and u be the solution of

$$\triangle u = 0 \ \text{ in } \ D$$

$$u(x) = g(x) \ \text{ on } \ \partial D.$$

Use the representation

$$u(x) = \frac{1}{2\pi} \int_{\partial D} \left[\ln \left(\frac{1}{|y-x|} \right) \ \partial_n u(y) - u(y) \partial_n \ln \left(\frac{1}{|y-x|} \right) \right] ds$$

to obtain the Green's function of the above problem.

8. Derive the Laplace's equation in polar coordinates.