Math 470- Exam 2

ID. Num.: Name:

<u>**Question 1**</u>: Prove that

$$u(x,y) = \begin{cases} \frac{1}{2}(f(x-y) + f(x+y)) + \frac{1}{2}\int_{x-y}^{x+y} g(t)dt & : x \geq y \\ \\ h(y-x) + \frac{1}{2}(f(x+y) - f(y-x)) + \frac{1}{2}\int_{y-x}^{y+x} g(t)dt & : x < y \end{cases}$$

is a solution of the problem

$$\begin{aligned} -u_{xx} + u_{yy} &= 0, \quad x > 0, \quad y > 0 \\ u(x,0) = f(x), \quad u_y(x,0) &= g(x), \quad x \geq 0 \\ u(0,y) &= h(y), \quad y \geq 0. \end{aligned}$$

Question 2: Solve the following IBVP using a suitable change of variables.

$$
\begin{aligned}
-u_{xx} + u_{yy} &= e^x, \qquad x > 0, \quad y > 0 \\
u(x,0) = a, \quad u_y(x,0) &= b, \qquad x \geq 0 \\
u(0,y) &= y, \qquad y \geq 0,
\end{aligned}
$$

where $a, b \in R$.

Question 3: Let $u(x,t) = f(\psi(x), t)$ be the solution of

$$u_{tt} = \Delta u, \quad x \in R^3, \quad t \geq 0$$

$$u(x,0) = 0, \quad u_t(x,0) = \psi(x), \quad x \in R^3.$$

Find and also **verify** the solution of

$$w_{tt} = \Delta w, \quad x \in R^3, \quad t \geq 0$$

$$w(x,0) = \varphi(x), \quad w_t(x,0) = \psi(x), \quad x \in R^3,$$

where φ and ψ are twice continuously differentiable functions.

Question 4: Prove that the solution of the following problem is unique.

$$-u_{xx} + u_y = e^{xy}, \quad x \in (0, \pi), \quad y > 0$$

$$u(x, 0) = \cos x, \quad x \in [0, \pi]$$

$$u(0, y) = e^y, \quad u(\pi, y) = \sin y, \quad y \geq 0.$$

Question 5: Let $a,\ b \in R$, and f be a smooth function. Let also $u = h(x,t)$ be the solution of

$$u_t = u_{xx}, \quad x \in (0, L), \quad t > 0$$

$$u(0,t) = 0, \quad u_x(L,t) = 0, \quad t \geq 0; \ u(x,0) = f(x) - a - bx, \quad x \in [0, L].$$

Solve the initial boundary value problem:

$$w_t = w_{xx}, \quad x \in (0, L), \quad t > 0$$

$$w(0,t) = a, \quad w_x(L,t) = b, \quad t \geq 0; \ w(x,0) = f(x), \quad x \in [0, L].$$