Math 470- Exam 1

ID. Num.: Name: Sec. Num.:

Q 1: Let $u = u(x, t)$. Find the general solution of

$$3u_x + 5u_t - xtu = 0.$$

Q 2: Use the method of characteristics to solve

$$\begin{aligned} uu_x + u_y &= 0, \\ u(x,0) &= g(x) \end{aligned}$$

Q 3: Use characteristics to solve

$$u_{xx} + 2u_{xy} - 3u_{yy} = 0.$$

Q 4: Derive the canonical form of the partial differential equation:

$$u_{xx} + 2u_{xy} + 5u_{yy} + u_y = 0.$$

Q 5: Use the d'Alembert's formula to derive the solution of the wave equation in the half-line:

$$\begin{aligned} u_{tt} &= u_{xx}, \quad x > 0 \quad t > 0 \\ u(x,0) &= f(x), \quad u_t(x,0) = g(x), \quad x \geq 0 \\ u(0,t) &= 0, \quad t \geq 0. \end{aligned}$$