King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics Sciences
Math 425 - Graph Theory
Semester –151

Final Exam | **Dr. M. Z. Abu-Sbeih** | **December 21, 2015**

Student No.: __________ Name: ___________________________________

Show all your work. No credits for answers without justification.
Write neatly and eligibly. You may loose points for messy work.

Problem 1 (10 points): Define each of the following

(a) Strong tournament
(b) Perfect matching
(c) Outer planar graph
(d) A t-tough graph
(e) The line graph $L(G)$ of a graph G

Problem 2 (16 points):

a) State the Max-Flow Min-Cut Theorem
b) The plane graph *G* has order 10 and size 20. The dual graph *G** has:
Order ______________ Size _____________ Number of regions___________
c) Draw a caterpillar *T* of order 7. Number the vertices randomly from 1 to 7, and then find the Prufer sequence of *T*.
d) Draw a tree *T* whose Prufer sequence is (4 4 4 5 5 5).

Problem 3 (25 points): Let $G = K_{r,r}$ where $r \geq 2$

(a) Find the number of distinct labeling of *G*. (5 points)
(b) Find the number of spanning trees of *G*. (5 points)
(c) Is *G* vertex-transitive graph? Why? (5 points)
(d) For what values of *r* the graph *G* is:
  i) Eulerian: ________________________
  ii) Hamiltonian ________________________
  iii) Planar ________________________
  iv) 1-tough ________________________
  v) 2-factorable ________________________

(e) Find each of the following:
  i) $k(G) =$ ________________________
  ii) Connectivity $\kappa(G) =$ ________________________
  iii) Independence number $\alpha(G) =$ ________________________
  iv) The girth $g(G) =$ ________________________

v) Edge connectivity $\lambda(G) =$ ________________________

Problem 4 (12 points): Prove each of the following

(a) Prove that every r-regular bipartite graph, $r \geq 1$, is 1-factorable.

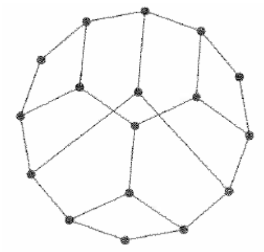
(b) Show that the cube Q_n is 1-factorable for all $n \geq 1$.

Problem 5 (12 points):

(a) Let G be a connected planar graph of order $n (n \geq 5)$ and size m whose shortest cycle length is 5. Prove that $m \leq \frac{5}{3}(n-2)$.

(b) Show that the Petersen graph is not planar.

Problem 6 (12 points):



(a) Determine the crossing number of $K_{1,2,3}$

(b) Show that the graph in the figure is not Hamiltonian.

Problem 7 (12 points):

(a) Let G be a bipartite graph with bipartition (U, W) such that $|U| = |W| = n \geq 2$. Let G' be the graph obtained from G by adding edges so that $\langle U \rangle$ is complete; i.e. $\langle U \rangle = K_n$. Prove that if G' is Hamiltonian then G is Hamiltonian.

(b) Prove that if G is Hamiltonian connected of order $n \geq 4$, then G is 3-connected.

Problem 8 (40 points): Let G be a graph of order $n \geq 5$ such that $d(v) \geq \frac{n-1}{2}$ for each vertex v of G. Either prove or disprove each of the following

(a) G is connected.

(b) G is nonseparable.

(c) G has a cycle of length at least $\frac{n+1}{2}$.

(d) G is Hamiltonian.

(e) G is Eulerian.

(f) G is panconnected.

(g) G has a perfect matching.

(h) $\kappa(G) = \delta(G)$