King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics Sciences
Math 425 - Graph Theory
Semester – 151

Exam III **Dr. M. Z. Abu-Sbeih** **December 7, 2015**

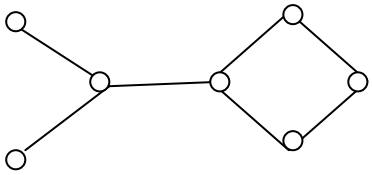
Student No.: _____. Name: _____

Show all your work. No credits for answers without justification.
Write neatly and eligibly. You may loose points for messy work.

Problem 1 (25 points):

(a) Give an example of a planar graph which isomorphic to its dual graph.

(b) Find the number of distinct labeling of the graph in the figure.



(c) Give a maximal planar graph of order 6

Problem 3 (25 points): *Either prove or disprove each of the following statements. If a statement is true sketch the proof, and if it is false, give a counter example.*

(a) Every induced subgraph of the complete graph K_n is complete.

$G = K_4 - e$

(b) If *k* is an odd integer and *G* is a *k*-regular graph of size m, then *m* is a multiple of *k*.

(c) If G_1 and G_2 are regular graphs, then the join $G_1 \vee G_2$ is regular.

(d) If the graph *G* has only two vertices of odd degree, then they must be connected by a path.

(e) Any connected graph has only one central vertex.

Problem 4 (29 points):

1) Let *G* is a graph of order $2n$ and size *m*. If $\delta(G) \geq n$ for each vertex *v*, prove that G is connected.

2) Prove that if *G* is an acyclic graph of order *n* and size *m* such that $m = n - 1$, then *G* is a tree.

3) Let G be a connected graph of order n $(n \geq 3)$. Prove that there is an orientation of G in which no directed path has length 2 if and only if G is bipartite.

4) Apply Kruskal's algorithm to find a minimum spanning tree T in the weighted graph G. Show how this tree is constructed. Also find $w(T)$.