King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics Sciences
Math 425 - Graph Theory
Semester – 151

Exam II | **Dr. M. Z. Abu-Sbeih** | **November 9, 2015**

Student No.: __________. Name: ___________________________________

Show all your work. No credits for answers without justification.
Write neatly and eligibly. You may loose points for messy work.

Problem 1 (21 points):

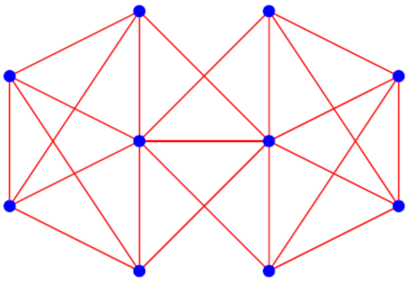
(a) Define Hamiltonian graph

(b) Give five conditions on the graph G (of order $n \geq 2$), each of which implies that G is Hamiltonian.

(c) Determine the connectivity and the edge-connectivity of the graph in the picture.

(d) Is the graph Eulerian? Why?

(e) Is the graph Hamiltonian? Why?



Problem 2 (15 points):

(a) State Menger's Theorem.

(b) Show that $\kappa(Q_n) = \lambda(Q_n) = n$ for $n \geq 2$.

Problem 3 (36 points): *Determine whether each of the following statements is true or false. If a statement is true sketch the proof, and if it is false, give a counter example.*

(a) If G_1 and G_2 are Eulerian, then $G_1 \vee G_2$ is Eulerian.

(b) If a graph G is pancyclic, then G is panconnected.

(c) Let G be nontrivial graph and $v \in V(G)$. Then $\kappa(G-v) = \kappa(G)$ or $\kappa(G-v) = \kappa(G) - 1$.

(d) Assume that $G = K_{m,n}$ $(m \geq n \geq 2)$ has odd degree, then G is Hamiltonian.

(e) If G is Eulerian, then $L(G)$ is Eulerian too.

(f) No bipartite graph is Hamiltonian-connected.

Problem 4 (28 points): Short proofs. Prove each of the following

1) If T is a tree containing at least one vertex of degree 2, then $\overline{T}$ is not Eulerian.

2) If $d(v) \geq \frac{n}{2}$ for each vertex v of a graph G of order $n \geq 3$, then $k(G-S) \leq |S|$ for each nonempty proper subset S of $V(G)$.

3) Every 1-tough graph is 2-connected.

4) If G is nontrivial connected graph, then $T(G^2)$ is Hamiltonian.