King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics Sciences
Math 425 - Graph Theory
Semester – 151

Exam I **Dr. M. Z. Abu-Sbeih** **October 14, 2015**

Student No.: __________. Name: ___________________________________

Show all your work. No credits for answers without justification.
Write neatly and eligibly. You may loose points for messy work.

Problem 1 (21 points): *Define each of the following*

(a) Nonseparable graph

(b) Isomorphic graphs

(c) Eccentricity

(d) Center of a graph

(e) Diameter of a graph

(f) The Matrix-Tree Theorem.

(g) Cayley's Tree Theorem.

Problem 2 (25 points):

(a) Give an example of a nontrivial self-complementary graph.

(b) What is the maximum number of cut-vertices in a connected graph of order n $(n \geq 3)$? How many bridges such a graph has?

(c) State Havel-Hakimi Theorem. Use to determine whether the sequence is graphic or not: (2,2,3,4,5,6,6).

(d) Determine the labeled tree with Prufer sequence: (3,3,1,6,2,2).

(e) Let $G = K_4 - e$ and let $A = \left[a_{ij}\right]$ be the adjacency matrix of G. Without finding the matrix, find $\sum_{i=1}^{4} a_{ii}^{(2)}$ and $\sum_{i=1}^{4} a_{ii}^{(3)}$.

<u>Problem 3 (25 points):</u> *Either prove or disprove each of the following statements. If a statement is true sketch the proof, and if it is false, give a counter example.*

(a) Every induced subgraph of the complete graph K_n is complete.

(b) If k is an odd integer and G is a k-regular graph of size m, then m is a multiple of k.

(c) If G_1 and G_2 are regular graphs, then the join $G_1 \vee G_2$ is regular.

(d) If the graph G has only two vertices of odd degree, then they must be connected by a path.

(e) Any connected graph has only one central vertex.

Problem 4 (29 points):

1) Let G is a graph of order $2n$ and size *m*. If $\delta(G) \geq n$ for each vertex *v*, prove that G is connected.

2) Prove that if G is an acyclic graph of order n and size m such that $m = n - 1$, then G is a tree.

3) Let G be a connected graph of order n $(n \geq 3)$. Prove that there is an orientation of G in which no directed path has length 2 if and only if G is bipartite.

4) Apply Kruskal's algorithm to find a minimum spanning tree T in the weighted graph G. Show how this tree is constructed. Also find $w(T)$.