King Fahd University of Petroleum & Minerals
Department of Mathematics and Statistics
MATH 321-01(Term 151)
Final Exam
December 22, 2015

NAME: ....

ID #: . . . .

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Question	Points	Score
1	14	
2	14	
3	15	
4	16	
5	16	
6	16	
7	14	
8	14	
9	14	
10	14	
11	14	
12	14	
Total	175	

Q1. Discuss briefly the two principal contributions to total numerical error in computational approximations. From what do each of these arise, i.e., what is the source of each?

Q2. Explain, in words, the difference between direct methods and iterative methods for solving an $n \times n$ system of linear equations. Name a method for each type.

Q3. (i) Why do wee need a stopping criteria for a numerical method?

(ii) Suppose we are calculating the root of a nonlinear equation $f(x) = 0$ using a numerical method and x^k is the iterate at the k^{th} step with TOL is a prescribed tolerance. Discuss each of the following stopping criteria. Which do you recommend using and why?

(i) $f(x^k) \leq TOL$
(ii) $|f(x^k)| \leq TOL$
(iii) $|x^{k+1} - x^k| \leq TOL$
(iv) $\frac{|x^{k+1} - x^k|}{|x^{k+1}|} \leq TOL$

Q4. Show how Gaussian elimination with **partial pivoting** work on

$$A = \begin{bmatrix} 2 & 3 & -4 & 1 \\ 1 & -1 & 0 & -2 \\ 3 & 3 & 4 & 3 \\ 4 & 1 & 0 & -4 \end{bmatrix}$$

Show all intermediate matrices.

Q5. Find $x_0,\ x_1$, and c_1 such that

$$\int_0^1 f(x)dx = \frac{1}{2}f(x_0) + c_1 f(x_1)$$

is exact for polynomials of degree less than or equal to 2.

Is the resulting quadrature rule exact for polynomials of degree less than or equal to 3?

Q6. Determine all the values of a, b, c, d, e, and f for which the following is a cubic spline:

$$s(x) = \begin{cases} ax^2 + b(x-1)^3 & x \in (-\infty, 1] \\ cx^2 + d & x \in [1, 2] \\ ex^2 + f(x-2)^3 & x \in [2, \infty) \end{cases}$$

Q7. Derive the approximation formula

$$f'(x) \approx \frac{f(x+h) - f(x-h)}{2h}$$

and determine the simplified form of its error term

Q8. Let $f(x) = x \ln x$. Determine the value of n (number of points) that will ensure an approximation error of less than 0.00002 when approximating $\int_1^2 f(x)\, dx$ using composite Simpson's rule:

$$\int_a^b f(x)\, dx = \frac{h}{3}[f(a) + 2 \sum_{j=1}^{(n/2)-1} f(x_{2j}) + 4 \sum_{j=1}^{n/2} f(x_{2j-1}) + f(b)] - \frac{b-a}{180} h^4 f^{(4)}(\mu)$$

Use that n to list the nodes and set up the rule (Do not evaluate).

Q9. Consider a linear system $Ax = b$. Derive the matrix form of the Jacobi iterative technique $x^{(k)} = Tx^{(k-1)} + c$.

Q10. Find the first two iterations of the GUSS-Seidel method for the following linear system, using $x^{(0)} = [0, 0, 0]^t$:

$$\begin{aligned} 4x_1 + x_2 - x_3 &= 5 \\ -x_1 + 3x_2 + x_3 &= -4 \\ 2x_1 + 2x_2 + 5x_3 &= 1 \end{aligned}$$

Use three-digit rounding arithmetic.

Q11. What is the total error that is needed to be minimized in the general problem of fitting the best least squares line to a collection of data $\{(x_i, y_i)\}_{i=1}^m$ and what do we need for a minimum to occur?

Q12. What property must a function $g(x)$ has to guarantee that the fixed point iteration $x_{n+1} = g(x_n)$ converges for all initial guesses x_0?