King Fahd University of Petroleum & Minerals
Department of Mathematics and Statistics
MATH 321-01(Term 151)
Exam II
November 26, 2015

NAME: .....

ID #: . . . . . .

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Question	Points	Score
1	14	
2	14	
3	14	
4	14	
5	16	
6	14	
7	14	
Total	100	

Q1. Consider the following integration formula

$$\int_{-1}^{1} f(x)dx = f(\alpha) + f(-\alpha)$$

For what value(s) of α, if any, will this formula be exact for all polynomials of degree 3 or less?

Q2. If we approximate an integral by Simpson's method, first using a step length h and then $h/2$ (for small h), how much do we expect the latter result to be more accurate?

Q3. (i) Let $f(t, y)$ be defined on a set $D \subset \mathbb{R}^2$. What do we mean when we say that f satisfies Lipschitz condition in the variable y.

(ii) Show that if $f(x)$ is Lipschitz with constant L and $|f(x)| \leq B$, then $[f(x)]^2$ is Lipschitz with constant $2LB$

Q4. Consider the following initial-value problem: $y' = \frac{1}{1+t^2} - 2y^2, \quad 0 \leq t \leq 1, \quad y(0) = 0,$ where the true solution is $y(t) = \frac{t}{1+t^2}$

Use Euler's method with $h = 0.25$ to approximate the solution

Q5. Determine a factorization in the form $A = P^t LU$ for the matrix

$$A = \begin{bmatrix} 1 & 2 & -4 & 3 \\ 2 & 5 & -6 & 10 \\ -2 & -7 & 3 & -21 \\ 2 & 8 & 15 & 38 \end{bmatrix}$$

and use it to solve $Ax = b$ for

$$b = \begin{bmatrix} 0 \\ 9 \\ -28 \\ 42 \end{bmatrix}$$

Q6. Show that the initial-value problem $y' = t^2 y + 1, \quad 0 \leq t \leq 1, \quad y(0) = 1$, is well-posed

Q7. Use Simpson's rule to approximate the following integral and find a bound for the theoretical error

$$\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \sin^2 x \, dx$$