King Fahd University of Petroleum & Minerals
Department of Mathematics and Statistics
MATH 321-01(Term 151)
Exam I
October 20, 2015

NAME: .....

ID #: . . . . . .

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Question	Points	Score
1	20	
2	12	
3	12	
4	15	
5	15	
6	13	
7	13	
Total	100	

Q1. Consider a smooth function f with the following values:

x	-1	0	1	2
f(x)	-1	0	3	-1

a) Write the polynomial interpolating $f(x)$ at the first 3 nodes in Lagrange form.

b) Use divided differences to find the interpolating polynomial $P(x)$ of all of the above data.

c) Assume all derivative of f are available, give an expression of the interpolating error $f(x) - P(x)$ for some $x \in [-1, 2]$

d) Evaluate $P(x)$ at $x = 1.05$ using: 3-digit chopping.

Q2. The equation $4x^2 - e^x - e^{-x} = 0$ has two positive solutions. Use two steps of the Newton's method to approximate one of these solutions starting with $p_0 = 5$. Use 4-digit rounding.

Q3. (i) Derive the Secant method

(ii) In general, which is faster, Newton's method or the Secant method?

Q4. Find the root of $x - e^{-x} = 0$ on the interval $[0.2, 1]$ accurate to within 10^{-3} using fixed-point iteration method for an appropriate iteration function $g(x)$ starting with $p_0 = 0.8$.

Estimate the number of iterations necessary to obtain the root and compare this theoretical estimate to the number actually needed.

Q5. Let f be defined at $a = x_0 < x_1 < x_2 < x_3 = b$.

Write the piecewise cubic spline $S(x)$ indicating the different boundary conditions, the known constants and those need to be determined.

Q6. Use the appropriate three-point formula to determine $f'(x)$ at the three given numbers

x	1.1	1.2	1.3
f(x)	9.025013	11.02318	13.46374

Q7. Let $f(x)$ be a function of x and $x_0, \ldots, x_n$ be $n+1$ distinct nodes. For $j = 0, \cdots, n$, let p_j be the polynomial interpolating f at the nodes $x_0, x_1, \cdots, x_j$. Let q be the polynomial interpolating f at the nodes $x_1, \cdots, x_n$. Show that:

$$p_n(x) = q(x) + \frac{x - x_n}{x_n - x_0}(q(x) - p_{n-1}(x))$$

Hint: compare the polynomials at the nodes.