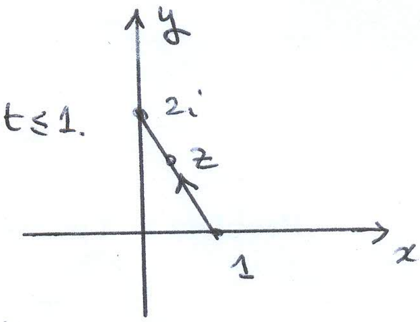
Math 302
Any answer without justification worths nothing

Quiz 6

Name: **ID #**

Problem 1 (3 points): Compute $\int_C |z^2|dz$, if C is the line segment joining $z_1 = 1$ and $z_2 = 2i$



Sol.

$$z - 1 = t(2i-1) \iff z = 1 + (2i-1)t, \quad 0 \le t \le 1.$$

$$= (1-t) + 2it$$

$$z = x + iy \Rightarrow z^2 = x^2 - y^2 + 2ixy$$

$$\Rightarrow |z^2| = |z|^2 = x^2 + y^2$$

$$= (1-t)^2 + (2t)^2 = 5t^2 - 2t + 1$$

$$\therefore \int_C |z^2| dz = \int_0^1 (5t^2 - 2t + 1)(2i-1)\, dt$$

$$= (2i-1) \left. \frac{5t^3}{3} - t^2 + t \right]_0^1$$

$$= (2i-1)\left(\frac{5}{3} - 1 + 1\right)$$

$$= \frac{5(2i-1)}{3}.$$

Problem 2 (3 points) Use the Cauchy formula to compute

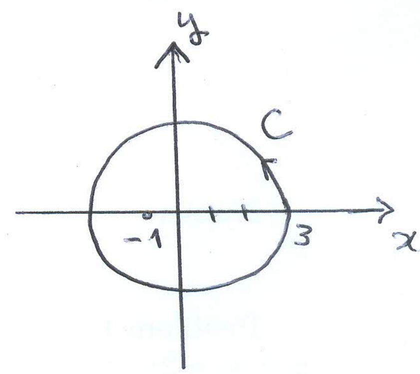
$$I = \oint_C (z^3 + \frac{i}{z+1})dz,$$

where $C : |z| = 3$, positively oriented.

$$I = \cancel{\oint_C z^3 dz}^{\,0} + i \oint_C \frac{dz}{z+1}$$

by Cauchy

$$= i\,(2\pi i)(1) = -2\pi.$$



Problem 3 (4 points) Use the "higher-order" Cauchy formula to compute

$$I = \oint_C \frac{dz}{z^2(z^2+1)},$$

where $C : |z - i| = \frac{3}{2}$, positively oriented.

$$I = \oint_C \frac{dz}{z^2(z-i)(z+i)}$$

$$= \oint_{\gamma_1} \frac{\frac{1}{z^2+1}}{z^2}\,dz + \oint_{\gamma_2} \frac{\frac{1}{z^2(z+i)}}{z-i}\,dz$$

$$= 2\pi i \,\frac{d}{dz}\left(\frac{1}{z^2+1}\right)\Big|_{z=0} + \cancel{2}\pi \cancel{i}\,\frac{1}{i^2(\cancel{2i})}$$

$$= 2\pi i \,\cancel{\frac{-2z}{(z^2+1)^2}\Big|_{z=0}}^{\,0} + (-\pi) = -\pi$$