Math 302

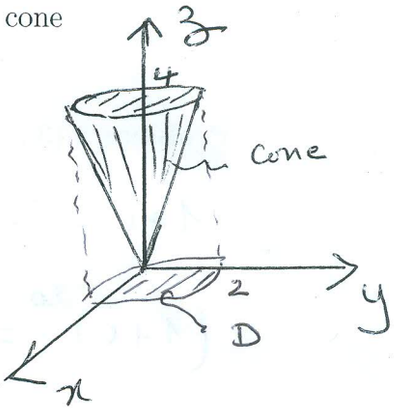
Any answer without justification worths nothing

Quiz 3

22/ 11/ 2015

Name: **ID #**

Problem 1 (4 points): Evaluate the flux $\iint_S F.ndS$ of the field $F = xy^2\,\vec{i} + x^2y\,\vec{j} + e^y \sin x^2\,\vec{k}$ through the surface given by the cone $z = \sqrt{x^2+y^2}$ and the plane $z = 4$.

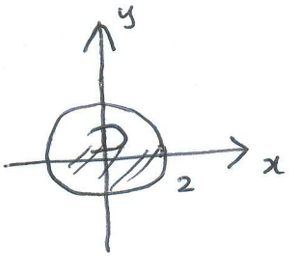


$$\iint_S F.\eta\, ds = \iiint_M \operatorname{div} F\, dV$$

$$= \iiint_M (y^2+x^2)\, dV$$

$$= \iint_D \int_{\sqrt{x^2+y^2}}^{4} \left((x^2+y^2)\, dA \right) dz$$

$$= \iint_D (x^2+y^2)\left(4 - \sqrt{x^2+y^2}\right) dA$$



$$= \int_0^{2\pi} \int_0^2 r^2(4-r)\, r\, dr\, d\theta = 2\pi \int_0^2 (4r^3 - r^4)\, dr$$

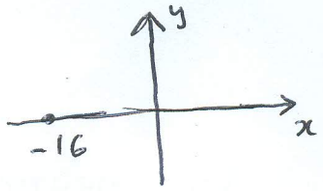
$$= 2\pi \left[r^4 - \frac{r^5}{5} \right]_0^2 = 2\pi \left(16 - \frac{32}{5}\right)$$

$$= \frac{96\pi}{5}.$$

Problem 2 (3 points) Compute all numbers $w = \sqrt{-16}$

$$-16 = 16(\cos\pi + i\sin\pi)$$

$$w = 4\left(\cos\frac{\pi+2k\pi}{2} + i\sin\frac{\pi+2k\pi}{2}\right), \quad k=0,1$$



$k=0$, $w_0 = 4\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right) = 4i$

$k=1$, $w_1 = 4\left(\cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2}\right) = -4i$

Problem 3 (3 points) Find $(1+i)^{20}$.

$$1+i = \sqrt{2}\left(\frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}}\right) = \sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$$

$$(1+i)^{20} = (\sqrt{2})^{20}\left(\cos\frac{20\pi}{4} + i\sin\frac{20\pi}{4}\right)$$

$$= 2^{10}(\cos 5\pi + i\sin 5\pi) = -2^{10}.$$