Math 302

Any answer without justification worths nothing

Quiz 3

25/ 10/ 2015

Name: **ID #**

Problem 1 (3 points): Find the length of the line given by

$$C = \{(\cos 2t, \frac{\sqrt{3}}{2}\sin 2t, \sin t\cos t), \qquad 0 \leq t \leq \pi\}.$$

length of C is

$$l = \int_0^\pi |r'(t)|\,dt, \quad r'(t) = \langle -2\sin 2t, \sqrt{3}\cos 2t, \cos 2t \rangle$$

so $$l = \int_0^\pi \sqrt{4\sin^2 2t + 3\cos^2 2t + \cos^2 2t}\,dt$$

$$= \int_0^\pi 2\,dt = 2\pi .$$

Problem 2 (4 points) Show that the tangent plane to the surface $z^2 = 2x^2 + 3y^2$ at the point $P_0 = (1, -1, \sqrt{5})$ passes through the original. That is the equation of the plane has the form $ax + by + cz = 0$.

The surface is given by $z^2 - 2x^2 - 3y^2 = F(x,y,z) = 0$

The normal is $N = \nabla F = \langle -4x, -6y, 2z \rangle$

P_0 is on the surface since $(\sqrt{5})^2 = 2 + 3$.

Normal at P_0 is $N_0 = \langle -4, 6, 2\sqrt{5} \rangle$

Equation of the plane at P_0 is

$\langle -4, 6, 2\sqrt{5} \rangle \cdot \langle x-1, y+1, z-\sqrt{5} \rangle = 0$

$-4(x-1) + 6(y+1) + 2\sqrt{5}(z - \sqrt{5}) = 0$

$-4x + \cancel{4} + 6y + \cancel{6} + 2\sqrt{5}\, z - \cancel{10} = 0$

$\Rightarrow \boxed{-2x + 3y + \sqrt{5}\, z = 0}$

Problem 3 (3 points) Find the directional derivative of $f(x, y) = x^2 - xy$ at the point $P_0 = (1, -2)$ in the direction of $u = \langle -\sqrt{3}, 1 \rangle$.

$|u| = 2 \Rightarrow \tilde{u} = \langle -\frac{\sqrt{3}}{2}, \frac{1}{2} \rangle$ is a unit vector.

$\nabla f(x,y) = \langle 2x - y, -x \rangle$ is continuous in $\mathbb{R}^2$

So

$D_u f(1,-2) = D_{\tilde{u}} f(1,-2) = \nabla f(1,-2) \cdot \langle -\frac{\sqrt{3}}{2}, \frac{1}{2} \rangle$

$= \langle 4, -1 \rangle \cdot \langle -\frac{\sqrt{3}}{2}, \frac{1}{2} \rangle$

$= -2\sqrt{3} - \frac{1}{2}$.