Math 302

Quiz 2

8/ 10/ 2015

Name: **ID #**

Given the matrix

$$A = \begin{pmatrix} 2 & -1 & 0 \\ -5 & 2 & 4 \\ 0 & 1 & 2 \end{pmatrix}$$

i) Find the inverse

$$\left(\begin{array}{ccc|ccc} 2 & -1 & 0 & 1 & 0 & 0 \\ -5 & 2 & 4 & 0 & 1 & 0 \\ 0 & 1 & 2 & 0 & 0 & 1 \end{array}\right) \xrightarrow{R_1/2} \left(\begin{array}{ccc|ccc} 1 & -\frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ -5 & 2 & 4 & 0 & 1 & 0 \\ 0 & 1 & 2 & 0 & 0 & 1 \end{array}\right)$$

$$\xrightarrow{R_2+5R_1} \left(\begin{array}{ccc|ccc} 1 & -\frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & -\frac{1}{2} & 4 & \frac{5}{2} & 1 & 0 \\ 0 & 1 & 2 & 0 & 0 & 1 \end{array}\right)$$

$$\xrightarrow{-2R_2} \left(\begin{array}{ccc|ccc} 1 & -\frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 1 & -8 & -5 & -2 & 0 \\ 0 & 1 & 2 & 0 & 0 & 1 \end{array}\right)$$

$$\xrightarrow[R_3-R_2]{R_1+\frac{1}{2}R_2} \left(\begin{array}{ccc|ccc} 1 & 0 & -4 & -2 & -1 & 0 \\ 0 & 1 & -8 & -5 & -2 & 0 \\ 0 & 0 & 10 & -5 & -2 & 1 \end{array}\right)$$

$$\xrightarrow{R_3/10} \left(\begin{array}{ccc|ccc} 1 & 0 & -4 & -2 & -1 & 0 \\ 0 & 1 & -8 & -5 & -2 & 0 \\ 0 & 0 & 1 & -\frac{1}{2} & -\frac{1}{5} & \frac{1}{10} \end{array}\right)$$

$$\xrightarrow[R_2+8R_3]{\quad R_3 \quad}\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -4 & -9/5 & 2/5 \\ 0 & 1 & 0 & -9 & -\frac{18}{5} & 4/5 \\ 0 & 0 & 1 & -1/2 & -\frac{1}{5} & \frac{1}{10} \end{array}\right)$$

So $A^{-1} = \begin{pmatrix} -4 & -9/5 & 2/5 \\ -9 & -18/5 & 4/5 \\ -\frac{1}{2} & -1/5 & 1/10 \end{pmatrix}$

ii) Find the eigenvalues of A: $\lambda_1 \leq \lambda_2 \leq \lambda_3$.

$$\begin{vmatrix} 2-\lambda & -1 & 0 \\ -5 & 2-\lambda & 4 \\ 0 & 1 & 2-\lambda \end{vmatrix} = (2-\lambda)\left[(2-\lambda)^2 - 4\right] + (-5)(2-\lambda)$$

$$= (2-\lambda)\left[(2-\lambda)^2 - 9\right] = (2-\lambda)(5-\lambda)(-1-\lambda) = 0$$

$\Rightarrow \lambda = -1, 2, 5$ are the eigenvalues.

(iii) Find an eigenvector corresponding to λ_1.

$\boxed{\lambda_1 = -1}$

$$\left(\begin{array}{ccc|c} 3 & -1 & 0 & 0 \\ -5 & +3 & 4 & 0 \\ 0 & 1 & +3 & 0 \end{array}\right) \xrightarrow{R_1/3} \left(\begin{array}{ccc|c} 1 & -1/3 & 0 & 0 \\ -5 & +3 & 4 & 0 \\ 0 & 1 & +3 & 0 \end{array}\right)$$

$$\xrightarrow{R_2+5R_1} \left(\begin{array}{ccc|c} 1 & -1/3 & 0 & 0 \\ 0 & 4/3 & 4 & 0 \\ 0 & 1 & 3 & 0 \end{array}\right) \xrightarrow{R_2 \leftrightarrow R_3} \left(\begin{array}{ccc|c} 1 & -1/3 & 0 & 0 \\ 0 & 1 & 3 & 0 \\ 0 & 4/3 & 4 & 0 \end{array}\right)$$

$$\xrightarrow[R_3 - 4/3 R_2]{R_1 + \frac{1}{3} R_2} \left(\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right). \quad \text{So} \quad \begin{array}{l} x_1 = -x_3 \\ x_2 = -3x_3 \end{array}$$

$E = \begin{pmatrix} -1 \\ -3 \\ 1 \end{pmatrix}$ is an eigenvector.