Math 302

Quiz 1

15/ 9/ 2015

Name: ID #

Problem 1 (5 points): Let

$$X = \{a =< a_1, a_2, a_3, a_1 + a_3 > \ / \ a_1, a_2, a_3 \in \mathbb{R}\}$$

1) Show that X is a subspace of $\mathbb{R}^4$
2) Find a basis and dim$X$.

Sol.

(i) By taking $a_1 = a_2 = a_3 = 0$, we get $0 = <0,0,0,0> \in X$

(ii) let $a, b \in X$. Then

$$a + b = < a_1, a_2, a_3, a_1 + a_3 > + < b_1, b_2, b_3, b_1 + b_3 >$$

$$= < a_1 + b_1, a_2 + b_2, a_3 + b_3, (a_1 + b_1) + (a_3 + b_3) >$$

$$= < c_1, c_2, c_3, c_1 + c_3 > \in X.$$

(iii) let $a \in X$, $\alpha \in \mathbb{R}$. Then

$$\alpha a = < \alpha a_1, \alpha a_2, \alpha a_3, \alpha a_1 + \alpha a_3 > \in X$$

So X is a subspace of $\mathbb{R}^4$.

2) $a \in X \Rightarrow a = < a_1, a_2, a_3, a_1 + a_3 >$

$$= a_1 < 1, 0, 0, 1 > + a_2 < 0, 1, 0, 0 >$$
$$+ a_3 < 0, 0, 1, 1 >.$$

A basis is $\{ <1,0,0,1>, <0,1,0,0>, <0,0,1,1> \}$

$\dim X = 3$.

Problem 2 (5 points):

Use Gauss-Jordan elimination method to determine the value (s) of α,for which the system is consistent.

$$\begin{cases} x_1 - 2x_2 + x_3 = 1 \\ 2x_1 \qquad - x_3 = 7 \\ 3x_1 - 2x_2 \qquad = \alpha \end{cases}$$

Sol.

$$\left(\begin{array}{ccc|c} 1 & -2 & 1 & 1 \\ 2 & 0 & -1 & 7 \\ 3 & -2 & 0 & \alpha \end{array}\right) \xrightarrow[R_3 - 3R_1]{\substack{R_1 \\ R_2 - 2R_1}} \left(\begin{array}{ccc|c} 1 & -2 & 1 & 1 \\ 0 & 4 & -3 & 5 \\ 0 & 4 & -3 & \alpha - 3 \end{array}\right)$$

Clearly the system is consistent iff $\alpha - 3 = 5$

$\Longleftrightarrow \quad \alpha = 8$