King Fahd University of Petroleum and Minerals

Department of Mathematics and Statistics

Math 301 Final Exam

Semester (151)

Dec. 27, 2015 at 07:00-10:00 PM

Name: ....

I.D: .... Section: .... Serial: ....

Question	Points
1	/15
2	/22
3	/22
4	/22
5	/22
6	/15
7	/22
Total	/140

Question 1 (9+3+3 points)

Consider the following Sturm-Liouville problem.

$$x^2y'' + 5xy' + \lambda y = 0, \quad y(1) = 0, \; y(2) = 0.$$

a. Put the equation in the self-adjoint form.

b. Find out the weight function.

c. Write the orthogonality relation.

Question 2 (22 points)

The steady-state temperature in a square plate modeled by the following BVP. Use the method of separation of variables to find $u(x, y)$.

$$\begin{cases} \dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} = 0, \quad 0 < x < \pi, \quad 0 < y < \pi \\ u_x(0, y) = 0, \qquad u_x(\pi, y) = 0 \\ u(x, 0) = 0, \qquad u(x, \pi) = x \end{cases}$$

***CONT*...Q2**

Question 3 (22 points)

The displacement $u\ (r,t)$ of a circular membrane of radius $c = 3$ clamped along its circumference with initial displacement and initial velocity in **polar** coordinates modeled by

$$\begin{cases} \dfrac{\partial^2 u}{\partial r^2} + \dfrac{1}{r}\dfrac{\partial u}{\partial r} = \dfrac{\partial^2 u}{\partial t^2}, & 0 < r < 3, \quad t > 0, \\ u(3,t) = 0, & t > 0, \\ u(r,0) = 1, & \left.\dfrac{\partial u}{\partial t}\right|_{t=0} = \dfrac{1}{2}. \end{cases}$$

Using separation of variables to find $u\ (r,t)$. Note that u is bounded at $r = 0$.

CONT…Q3

Question 4 (22 points)

Using **Cylindrical** Coordinates, the steady-state temperature $u(r, z)$ in a circular cylinder subject to conditions given by the following system.

$$\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial z^2} = 0, \quad 0 < r < 2, \quad 0 < z < 4,$$

$$u(2, z) = 0, \qquad 0 < z < 4,$$

$$u(r, 0) = 0, \qquad u(r, 4) = c_0 \qquad 0 < r < 2.$$

Find $u(r, z)$. Note that u is bounded at $r = 0$.

CONT…Q4

Question 5 (22 points)

Using the Laplace Transform to find the displacement $u\,(x,\,t)$ of a very long string in the following model.

$$\frac{\partial^2 u}{\partial x^2} - 10 = \frac{\partial^2 u}{\partial t^2}, \quad x > 0, \quad t > 0,$$

subject to

$$\begin{cases} u(0,t) = 0, & \lim_{x \to \infty} \dfrac{\partial u}{\partial x} = 0, \quad t > 0, \\ u(x,0) = 0, & \left.\dfrac{\partial u}{\partial t}\right|_{t=0} = 0, \quad x > 0. \end{cases}$$

CONT…Q5

Question 6 (10+5 points)

a. Represent $f(x) = e^{-2x},\ x > 0$ by Fourier **cosine** integral.

b. Use the result in (a) to evaluate $\int_0^\infty \frac{\cos t}{4+t^2}\,dt$.

Question 7 (22 points)

Use the **Fourier sine** to find the temperature $u(x,t)$ in a semi-infinite rod modeled by

$$\begin{cases} k\dfrac{\partial^2 u}{\partial x^2} = \dfrac{\partial u}{\partial t}, & x > 0, \quad t > 0, \\ u(0,t) = 0, & t > 0, \\ u(x,0) = f(x) = \begin{cases} 1, & 0 < x < 1 \\ 0, & x > 1. \end{cases} \end{cases}$$

CONT…Q7