King Fahd University of Petroleum and Minerals

Department of Mathematics and Statistics

Math 301 Second Exam

Semester (151)

Nov. 11, 2015 at 05:15-07:15 PM

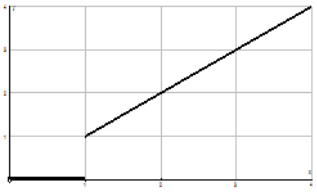
Name: .....

I.D: ..... Section: ..... Serial: .....

Question	Points
1	/15
2	/10
3	/20
4	/15
5	/10
6	/10
7	/20
Total	/100

Question 1 (8+7 points)

a. Use the definition of the Laplace transforms to find $\mathcal{L}\{f(t)\}$, where $f(t)$ is given in the adjacent figure



b. Use the method of partial fractions to find $\mathcal{L}^{-1}\left\{\frac{1}{s^3+5s}\right\}$.

Question 2 (5+5 points)

a) Use translation theorems to find $\mathcal{L}^{-1}\left\{\frac{5s}{(s-2)^2}\right\}$.

b) Write $f(t) = \begin{cases} 1 & , \ 0 \le t < 1 \\ -1 & , \ t \ge 1 \end{cases}$ in the compact form and find $\mathcal{L}\{f(t)\}$.

Question 3 (10+10 points)

a) Use the method of convolution to find $\mathcal{L}^{-1}\left\{\frac{3}{s^3-2s^2-9s+18}\right\}$.

b) Solve the following integrodifferential equation

$$y' = 1 - \sin t - \int_0^t y(s)ds, \qquad y(0) = 0.$$

Question 4 (15 points)

Use the Laplace transform to solve the following IVP

$$y'' - y' - 2y = \delta(t - \pi), \qquad y(0) = 1, \qquad y'(0) = 1.$$

Question 5 (5+5 points)

a) Show that the set $\{1, 2x\}$ is an orthogonal set with respect to the weight $\mathcal{W}(x) = e^{-x^2}$ over $(-\infty, \infty)$.

b) Let $\{\emptyset_n\}$ be a set of orthogonal functions over $[a, b]$. Show that

$$\|\emptyset_m + \emptyset_n\|^2 = \|\emptyset_m\|^2 + \|\emptyset_n\|^2, \qquad m \neq n \ .$$

Question 6 (10 points)

If the Fourier series of $f(x) = \begin{cases} 0 & , \ -\pi < x < 0 \\ \sin x & , \ 0 \le x < \pi \end{cases}$ is given by

$$f(x) = \frac{1}{\pi} + \frac{1}{2}\sin x + \sum_{n=2}^{\infty} \frac{1 + (-1)^n}{\pi(1 - n^2)} \cos nx \, .$$

Show that $\frac{\pi}{4} = \frac{1}{2} + \frac{1}{1\cdot 3} - \frac{1}{3\cdot 5} + \frac{1}{5\cdot 7} - \frac{1}{7\cdot 9} + \cdots$.

Question 7 (14+3+3 points)

Consider the function $f(x) = \begin{cases} 1 & , \ 0 < x < 1 \\ 2 - x & , \ 1 \leq x < 2 \end{cases}$.

a) Find the half-range **sine** expansions of $f(x)$.

b) Sketch the graph of the periodic extension of $f(x)$ on $(-6,6)$.

c) Find to what value this series converges when $x = 4$.