King Fahd University of Petroleum and Minerals

Department of Mathematics and Statistics

Math 301 First Exam

Semester (151) Oct. 14, 2015 at 06:00-08:00 PM

Name: ....

I.D: .... Section: .... Serial: ....

Question	Points
1	/10
2	/12
3	/15
4	/15
5	/13
6	/20
7	/15
Total	/100

Question 1 (5+5 points)

a. Let $\mathbf{r}(t) = t^2\mathbf{i} + t\mathbf{j} + \mathbf{k}$. Find $\frac{d}{dt}[\mathbf{r}(t) \times \mathbf{r}'(t)]$.

b. Find the length of the curve traced by

$$\mathbf{r}(t) = \frac{2\sqrt{2}}{3}t^{3/2}\mathbf{i} + t\cos t\,\mathbf{j} + t\sin t\,\mathbf{k}; \qquad 0 \leq t \leq \pi.$$

Question 2 (5+7 points)

a) Suppose $\nabla f(a, b) = 4\mathbf{i} + 3\mathbf{j}$. Find a unit vector $\mathbf{u}$ so that $D_{\mathbf{u}} f(a, b)$ is maximum.

b) Find an equation of the tangent plane to the graph of $xz = 6$ at the point $(2,0,3)$.

Question 3 (5+10 points)

a) For any constant vector **a** and $\mathbf{r}(t) = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$**,** show that $\nabla \cdot (\mathbf{a} \times \mathbf{r}) = 0$.

b) Evaluate $\int_C xy^2\, dy$, where ***C*** is the quarter-circle define by

$$x = 4\cos t,\;\; y = 4\sin t,\;\; 0 \le t \le \frac{\pi}{2}\,.$$

Question 4 (5+5+5 points)

Let $\mathbf{F}(x, y, z) = (y + yz)\mathbf{i} + (x + 3z^3 + xz)\mathbf{j} + (9yz^2 + xy - 1)\mathbf{k}$ be a field.

a) Show that $\mathbf{F}$ is conservative.

b) Show that $\phi(x, y, z) = xy + xyz + 3yz^3 - z$ is a potential of $\mathbf{F}$.

c) Evaluate $\int_{(1,0,1)}^{(4,1,2)} \mathbf{F} \cdot d\mathbf{r}$.

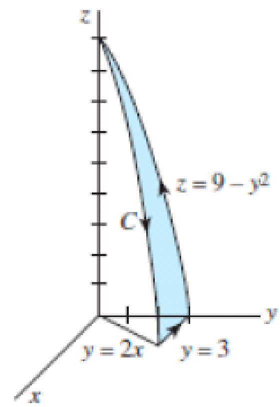
Question 5 (13 points)

Use Green's theorem to evaluate $\oint_C 2ydx + 5xdy$, where C is the circle

$$(x-1)^2 + (y+1)^2 = 25.$$

Question 6 (20 points)

Use Stoke's theorem to evaluate $\oint_C x^2y\,dx + (x + y^2)\,dy + xy^2z\,dz$, where **C** is the boundary of the surface shown in the adjacent figure.



Question 7 (10+5 points)

Use the divergence theorem to evaluate the outward flux, $\iint_S (\mathbf{F} \cdot \mathbf{n})dS$ where S is the surface of the sphere $x^2 + y^2 + z^2 = 9$ in the first octant and **n** is the outward unit normal vector to the surface. The vector field $\mathbf{F}$ is,

a) $\mathbf{F} = 4x\mathbf{i} + y\mathbf{j} + 4z\mathbf{k}$

b) $\mathbf{F} = y\cos 3z\ \mathbf{i} + ze^{-2xz}\mathbf{j} - \sin 2xy\,\mathbf{k}$