

King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics

Math 301 Second Exam

Semester (151)

Nov. 11, 2015 at 05:15-07:15 PM

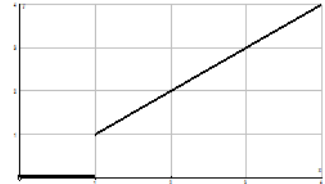
Name:

I.D: Section: Serial:

Question	Points
1	/15
2	/10
3	/20
4	/15
5	/10
6	/10
7	/20
Total	/100

Question 1**(8+7 points)**

a. Use the definition of the Laplace transforms to find $\mathcal{L}\{f(t)\}$, where $f(t)$ is given in the adjacent figure



b. Use the method of partial fractions to find $\mathcal{L}^{-1}\left\{\frac{1}{s^3+5s}\right\}$.

Question 2

(5+5 points)

a) Use translation theorems to find $\mathcal{L}^{-1}\left\{\frac{5s}{(s-2)^2}\right\}$.

b) Write $f(t) = \begin{cases} 1 & , 0 \leq t < 1 \\ -1 & , t \geq 1 \end{cases}$ in the compact form and find $\mathcal{L}\{f(t)\}$.

Question 3**(10+10 points)**

a) Use the method of convolution to find $\mathcal{L}^{-1}\left\{\frac{3}{s^3-2s^2-9s+18}\right\}$.

b) Solve the following integrodifferential equation

$$y' = 1 - \sin t - \int_0^t y(s)ds, \quad y(0) = 0.$$

Question 4**(15 points)**

Use the Laplace transform to solve the following IVP

$$y'' - y' - 2y = \delta(t - \pi), \quad y(0) = 1, \quad y'(0) = 1.$$

Question 5

(5+5 points)

a) Show that the set $\{1, 2x\}$ is an orthogonal set with respect to the weight

$$\mathcal{W}(x) = e^{-x^2} \text{ over } (-\infty, \infty) .$$

b) Let $\{\phi_n\}$ be a set of orthogonal functions over $[a, b]$. Show that

$$\|\phi_m + \phi_n\|^2 = \|\phi_m\|^2 + \|\phi_n\|^2, \quad m \neq n .$$

Question 6

(10 points)

If the Fourier series of $f(x) = \begin{cases} 0 & , -\pi < x < 0 \\ \sin x & , 0 \leq x < \pi \end{cases}$ is given by

$$f(x) = \frac{1}{\pi} + \frac{1}{2} \sin x + \sum_{n=2}^{\infty} \frac{1 + (-1)^n}{\pi(1 - n^2)} \cos nx .$$

Show that $\frac{\pi}{4} = \frac{1}{2} + \frac{1}{1 \cdot 3} - \frac{1}{3 \cdot 5} + \frac{1}{5 \cdot 7} - \frac{1}{7 \cdot 9} + \dots$.

Question 7**(14+3+3 points)**

Consider the function $f(x) = \begin{cases} 1 & , 0 < x < 1 \\ 2 - x & , 1 \leq x < 2 \end{cases}$.

- a) Find the half-range **sine** expansions of $f(x)$.
- b) Sketch the graph of the periodic extension of $f(x)$ on $(-6,6)$.
- c) Find to what value this series converges when $x = 4$.