King Fahd University of Petroleum & Minerals
Department of Mathematics & Statistics
Math-280, Term-151
Major Exam 2, Time Allowed: 2 hours

Name: **ID:**

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Question	Score	Total Mark
1		14
2		14
3		14
4		18
5		20
6		20
TOTAL		100

Question 1: Let V be a vector space. Show that

a) the element $\mathbf{0}$ in V is unique.

b) $\alpha\mathbf{0} = \mathbf{0}$ for each scalar α.

Question 2: Let S_1 and S_2 be two subspaces of a vector space V. Determine whether the following sets are subspaces of V.

a) $S_1 \cap S_2$

b) $S_1 \setminus S_2 = \{v \in S_1 : v \notin S_2\}$.

Question 3: Check for linear independence

a) $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & -1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ -1 & 0 \end{pmatrix}$ in $\mathbb{R}^{2\times 2}$.

b) $x_2 - x_1$, $x_3 - x_2$, $x_3 - x_1$ in $\mathbb{R}^n$ where x_1, x_2, x_3 are linearly independent vectors in $\mathbb{R}^n$.

Question 4:

a) Find a basis for the subspace S of $\mathbb{R}^4$ that consists of all vectors of the form $(a-2b, a-b-3c, b, a)^T$, where $a, b, c \in \mathbb{R}$. What is the dimension of S ?

b) In $C[-\pi, \pi]$, what is the dimension of $\text{Span}(1, \cos x, \sin^2(\frac{x}{2}))$.

Question 5: Let $\mathbf{u}_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$, $\mathbf{u}_2 = \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}$, $\mathbf{u}_3 = \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix}$.

a) Show that the set $B_1 = \{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ forms a basis for $\mathbb{R}^3$.

b) Find the transition matrix from the $B_2 = \{e_1, e_2, e_3\}$ to B_1.

c) Using part b) find $[v]_{B_1}$ where $v = (3, 2, -5)^T$.

Question 6: For the matrix

$$A = \begin{pmatrix} 3 & 1 & -3 & 4 \\ -1 & 2 & 1 & -2 \\ -3 & 8 & -4 & 2 \end{pmatrix}$$

find

a) a basis for

i. the row space of A,

ii. the column space of A, and

iii. the null space of A.

b) $\text{rank}(A)$, $\text{nullity}(A)$.