King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics

MATH 260 - Final Exam - Term 151

Duration: 180 minutes

Name:______________________________ ID Number:_____________________________

Section Number:_____________________ Serial Number:__________________________

Class Time: _________________________ Instructor's Name:___

Instructions:

1. Calculators and Mobile Phones are not allowed.
2. Write legibly.
3. For written questions, show all work. No points for answers without justification.
4. Make sure that you have 11 pages of problems (Total of 12 Problems)
   (There are 6 multiple choice and 6 written questions).
   Multiple choice questions carry 7 marks each.

Question Number	Points	Maximum Points
1		7
2		7
3		7
4		7
5		7
6		7
7		14
8		18
9		14
10		14
11		14
12		24
Total		140

1. The interval of validity for the IVP $(x^2-4)\,y'+3y=\ln|25-5x|$, $y(3)=2.5$ is:

   a) $(2<x<5)$
   b) $(-2<x<2)$
   c) $(-\infty<x<-2)$
   d) $(5<x<\infty)$
   e) $(2<x<\infty)$

2. A tea cup is taken out from an oven at $100^\circ\,C$. For tea to cool, it is left at a room temperature of $30^\circ\,C$. After 20 minutes, the temperature of tea is $80^\circ\,C$. The time tea will take to cool down to $40^\circ\,C$ is:

   a) $t=\dfrac{20\,\ln(1/7)}{\ln(5/7)}$
   b) $t=\dfrac{15\,\ln(1/7)}{\ln(5/3)}$
   c) $t=\dfrac{20\,\ln(1/7)}{\ln(3/7)}$
   d) $t=\dfrac{15\,\ln(3/7)}{\ln(5/7)}$
   e) $t=\dfrac{10\,\ln(4/7)}{\ln(6/7)}$

3. The general solution of the first order linear differential equation $(x^2+1)\,y' + 4xy = \dfrac{2x}{x^2+1}$ is:

a) $y = \dfrac{1}{(x^2+1)^2}\,[x^2 + C]$

b) $y = \dfrac{1}{(x^2+1)}\,[x^2 + C]$

c) $y = \dfrac{-1}{(x^2+1)}\,[x^2 + C]$

d) $y = \dfrac{1}{(x^2+1)^2}\,[2x^2 + C]$

e) $y = \dfrac{1}{x+1}\,\ln|x^2+1| + C$

4. A family of solutions of $(1 + x^2\,y + \cos x + (1/2)\,\ln y)\,dx + [(x/2y) + e^y + 3 + (x^3/3)]\,dy = 0$, with $y > 0$ is:

a) $f(x,y) = (x/2)\,\ln y + e^y + 3y + (x^3/3)\,y + x + \sin x = C$

b) $f(x,y) = (x/2)\,\ln y + 2e^y + 3y + (x^3/3)\,y + x + \sin x = C$

c) $f(x,y) = x\,\ln y + e^y + 3y + (x^3/3)\,y + x + \sin x = C$

d) $f(x,y) = (x/2)\,\ln y + 4e^y + 6y + (x^3/3)\,y + x + \sin x = C$

e) $f(x,y) = x\,\ln y + e^y + 3\ln y + (x^3/3)\,y + x + \sin x = C$

5. Using an appropriate substitution, we find that a family of solutions of $y' = \tan^2(x+y)$ is given by:

   a) $y = C + x - \sin(x+y)\,\cos(x+y)$
   b) $y = C - x - \sin(x+y)\,\cos(x+y)$
   c) $y = C + x + \sin(x+y)\,\cos(x+y)$
   d) $y = C - 2x - \sin(x+y)\,\cos(x+y)$
   e) $y = C + 2x + \sin(x+y)\,\cos(x+y)$

6. If y is a solution of the IVP $x\,y'\,e^{(y/x)} = x + y\,e^{y/x}$, $y(1) = 1$, then $y(e)$ is equal to:

   a) $e\,\ln(e+1)$
   b) $e + \ln(e+1)$
   c) $e\,\ln(e-1)$
   d) $e - \ln(e-1)$
   e) $e + \ln(e-1)$

7. **(14 points)** Use method of undetermined coefficients to find particular solution of $y'' + 4y = 2\ \cos 2x + e^x + 1$.

8. **(18 points)** Determine whether or not the matrix $A = \begin{pmatrix} 3 & -3 & 1 \\ 2 & -2 & 1 \\ 0 & 0 & 1 \end{pmatrix}$ is diagonalizable. If it is diagonalizable, find a diagonalizing matrix P and a diagonal matrix D. Use your results to calculate A^{50}.

9. **(14 points)** Find general solution of the system

$$X'(t) = \begin{pmatrix} -3 & 0 & -4 \\ -1 & -1 & -1 \\ 1 & 0 & 1 \end{pmatrix} X(t).$$

The egienvalues of the matrix are $\lambda = -1$ (multiplicity 3).

10. **(14 points)** Given that $y_1 = x$ and $y_2 = e^x$ are solutions of the DE $(1-x)\,y'' + xy' - y = 0$, use variation of parameters method to find a particular solution of $(1-x)\,y'' + xy' - y = 2(x-1)^2\,e^{-x}$.

11. **(14 points)** Solve the first order system

$$X'(t) = \begin{pmatrix} 1 & -1 & 2 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{pmatrix} X(t)$$

12. Consider the homogeneous system $X'(t) = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ -2 & 0 & -1 \end{bmatrix} X(t)$.

(a)**(9 points)** The vectors $X_1 = \begin{bmatrix} \cos t \\ \dfrac{-1}{2}\cos t + \dfrac{1}{2}\sin t \\ -\cos t - \sin t \end{bmatrix}$, $X_2 = \begin{bmatrix} 0 \\ e^t \\ 0 \end{bmatrix}$ and $X_3 = \begin{bmatrix} \sin t \\ \dfrac{-1}{2}\sin t - \dfrac{1}{2}\cos t \\ -\sin t + \cos t \end{bmatrix}$ are solutions of the system. Choose any two vectors to verify that they are solutions of the given system.

(b) **(6 points)** Determine whether the solutions in part (a) are linearly independent.

(c) **(3 points)** Write general solution of the given system.

(d) **(6 points)** Solve the IVP $X'(t) = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ -2 & 0 & -1 \end{bmatrix} X(t)$, with

$X(0) = \begin{bmatrix} x_1(0) \\ x_2(0) \\ x_3(0) \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}.$