KFUPM, DEPARTMENT OF MATHEMATICS AND STATISTICS

MATH 232: FINAL, SEMESTER (151), DECEMBER 28, 2015

08:00–11:00 am

Name : ....

ID : ....

Exercise	**Points**
1	: **10**
2	: **10**
3	: **10**
4	: **10**
5	: **14**
6	: **20**
7	: **13**
8	: **10**
9	: **10**
10	: **13**
11	: **7**
12	: **13**
Total	: **140**

Exercise 1 (10 pts)**.** Let P, Q, R be statements. Explain why the following argument is invalid:

$$\begin{array}{lcl} P & \longrightarrow & Q \\ R & \longrightarrow & Q \\ \hline \therefore P & \longrightarrow & R \end{array}$$

Exercise 2 (10 pts)**.** Find all the orderings $R \subseteq A \times A$ on the set $A = \{a, b, c\}$.

Exercise 3 (10 pts)**.** Let S be a set (not necessarily finite). Show that

$$\mid S \mid < \mid P(S) \mid .$$

Exercise 4 (10 pts)**.** Find all integers x, y such that
$$198x + 54y = 18.$$

Exercise 5 (14 pts)**.** Find all the positive integers a, b satisfying the following properties:

- $a < b$
- $\gcd(a, b) = 20$
- $\operatorname{lcm}(a, b) = 840$

Exercise 6 (20 pts)**.** Let R be the relation defined on the cartesian product $\mathbb{Z} \times (\mathbb{Z} - \{0\})$ by:

$$(x, y)R(a, b) \Longleftrightarrow xb = ya.$$

(1) Show that R is an equivalence relation.

(2) If $(x, y) \in \mathbb{Z} \times (\mathbb{Z} - \{0\})$, then we denote by $\overline{(x, y)}$ the equivalence class of (x, y) with respect to the equivalence relation R.

Show that if $d = \gcd(x, y)$ and $(x_1, y_1) = (\frac{x}{d}, \frac{y}{d})$, then $\overline{(x, y)} = \overline{(x_1, y_1)}$.

(3) Show that $\overline{(600, 420)} = \overline{(5, 7)}$.

Define the addition $\oplus$ on the quotient set $\Gamma := [\mathbb{Z} \times (\mathbb{Z} - \{0\})]/R$ by:

$$\overline{(x,y)} \oplus \overline{(a,b)} = \overline{(xb + ya, yb)},$$

and the multiplication $\otimes$ by:

$$\overline{(x,y)} \otimes \overline{(a,b)} = \overline{(xa, yb)}.$$

(4) Show that $(\Gamma, \oplus)$ is an Abelian group.

(5) Show that $(\Gamma - \{\overline{(0,1)}\}, \otimes)$ is an Abelian group.

Exercise 7 (13 pts)**.** Prove that the function $f : \mathbb{R} \setminus \{3\} \longrightarrow \mathbb{R} \setminus \{7\}$ defined by $f(x) = \dfrac{7x+11}{x-3}$ is bijective. Find its inverse function f^{-1}.

Exercise 8 (10 pts)**.** Let $\mathbb{N} = \{1, 2, 3, \ldots\}$. Give an explicit bijection from $\mathbb{N} \times \mathbb{N}$ onto $\mathbb{Z}$.

Exercise 9 (10 pts)**.** Give an explicit bijection from the open interval $(1, 2)$ onto $\mathbb{R}$.

Exercise 10 (13 pts)**.** List all the elements of S_3 and give its table of multiplication. Find all the subgroups of S_3.

Exercise 11 (7 pts)**.** Let G be a group and H be a subgroup of G. If $\mid G \mid= 60$, then what are all the possible values of $\mid H \mid$.

Exercise 12 (13 pts)**.** Let U be the set of all $x \in Z_{14}$ having an inverse for the multiplication.

(1) List all the elements of U.
(2) Give the table of multiplication on U.
(3) Find the order of each element of U.