KFUPM, DEPARTMENT OF MATHEMATICS AND STATISTICS

MATH 232: EXAM I, SEMESTER (151), OCTOBER 20, 2015

08:00–10:00 pm

Name : ....

ID : ....

Exercise	**Points**
1	: **9**
2	: **7**
3	: **5**
4	: **9**
5	: **9**
6	: **9**
7	: **6**
8	: **4**
9	: **9**
10	: **4**
11	: **9**
Total	: **80**

Exercise 1. Write the propositional form of the following argument. Then use algebraic properties of logical equivalence to show that it is valid (**no truth table will be accepted**).

Argument:

(1) If it is sunny tomorrow, then we will have a picnic
(2) If it rains, then we will go bowling.
(3) Either it will be sunny or it will rain tomorrow.
(4) Therefore, either we will have a picnic or we will go bowling.

Exercise 2. Write the propositional form of the following argument. Then use algebraic properties of logical equivalence to show that it is valid (**no truth table will be accepted**).

Argument:

(1) Either the meeting is in room 201, or it is in room 202.
(2) It is not in room 201.
(3) Therefore, it is in room 202.

Exercise 3. Explain why the following argument is a fallacy

Argument:

(1) All humans are mortal.
(2) Dogs are mortal.
(3) Therefore, humans are dogs.

Exercise 4. Let P, Q be two propositions. Show that

$$(P \implies Q) \vee (Q \implies P)$$

is a tautology.

Exercise 5. Let P, Q be two propositions. Use logical equivalences to show that

$$[(P \wedge \overline{(\overline{P} \vee Q)}) \vee (P \wedge Q)] \Longrightarrow P$$

is a tautology.

Exercise 6. Let $x \in \mathbb{Z}$. Prove by contrapositive that if $x^2 + 2x + 3$ is odd, then x is even.

Exercise 7. Using proof by cases, show that for all $a, b \in \mathbb{R}$, we have $|ab| = |a||b|$.

Exercise 8. Let $x \in \mathbb{R}$. Show that if $x^2 - 2x + 3 = 0$, then $e^{x^2} = e^{2x+3}$.

Exercise 9. Show that for all $n \in \mathbb{Z}$, if n and $n - 2$ are not divisible by 4, then $n^2 - 1$ is divisible by 4 (Hint: write the Euclidean division of n by 4).

Exercise 10. Show that for all $x, y \in \mathbb{R}$, the following inequality holds
$$\frac{9}{16}x^2 + y^2 \geq \frac{3}{2}xy.$$

Exercise 11. For each of the following quantified statements, decide whether it is true or false:

(1) $\forall x \in \mathbb{R}, \forall y \in \mathbb{R}, |x+y| > |x-y|$.

(2) $\exists x \in \mathbb{R}, \exists y \in \mathbb{R}, |x+y| > |x-y|$.

(3) $\forall x \in \mathbb{R}, \exists y \in \mathbb{R}, |x+y| > |x-y|$.

(4) $\exists x \in \mathbb{R}, \forall y \in \mathbb{R}, |x+y| > |x-y|$.