MATH 202.10 (Term 151)

Quiz 6 (Sects. 8.1 & 8.2) Duration: 20min

Name: ID number:

1.)(5pts)Solve the homogeneous linear system $X' = \begin{pmatrix} 4 & -1 \\ 1 & 6 \end{pmatrix} X$.

2.)(5pts)Solve the homogeneous linear system $X' = \begin{pmatrix} -6 & 1 \\ -2 & -4 \end{pmatrix} X$.

1.) $X' = AX$, $A = \begin{pmatrix} 4 & -1 \\ 1 & 6 \end{pmatrix}$

$\det(A - \lambda I) = \begin{vmatrix} 4-\lambda & -1 \\ 1 & 6-\lambda \end{vmatrix} = (4-\lambda)(6-\lambda)+1$
$= \lambda^2 - 10\lambda + 25$
$= (\lambda-5)^2$

$\lambda = 5$

$(A - 5I)K = 0$; $\left(\begin{array}{cc|c} -1 & -1 & 0 \\ 1 & 1 & 0 \end{array}\right), \left(\begin{array}{cc|c} 1 & 1 & 0 \\ 0 & 0 & 0 \end{array}\right)$

$x + y = 0 \Rightarrow K \begin{pmatrix} 1 \\ -1 \end{pmatrix}$

$X_1 = \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{5t}$

$X_2 = (tK + P)e^{5t}$, $(A - 5I)P = K$

$\left(\begin{array}{cc|c} -1 & -1 & 1 \\ 1 & 1 & -1 \end{array}\right), \left(\begin{array}{cc|c} 1 & 1 & -1 \\ 0 & 0 & 0 \end{array}\right)$

$x + y = -1 \Rightarrow P \begin{pmatrix} -1 \\ 0 \end{pmatrix}$

$\Rightarrow X_2 = \left[t\begin{pmatrix} 1 \\ -1 \end{pmatrix} + \begin{pmatrix} -1 \\ 0 \end{pmatrix}\right] e^{5t}$

$= \begin{pmatrix} t-1 \\ -t \end{pmatrix} e^{5t}$

$\Rightarrow \boxed{X = c_1 \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{5t} + c_2 \begin{pmatrix} t-1 \\ -t \end{pmatrix} e^{5t}}$,

$t \in (-\infty, \infty)$.

2.) $X' = AX$, $A = \begin{pmatrix} -6 & 1 \\ -2 & -4 \end{pmatrix}$

$\det(A - \lambda I) = \begin{vmatrix} -6-\lambda & 1 \\ -2 & -4-\lambda \end{vmatrix} = (-6-\lambda)(-4-\lambda) + 2$
$= \lambda^2 + 10\lambda + 26$

$\Delta = 100 - 104 = -4$

$\lambda = \dfrac{-10 \mp 2i}{2} = -5 \pm i$

$\lambda_1 = -5 + i$ $\quad (A - (-5+i)I)K_1 = 0$

$\left(\begin{array}{cc|c} -6-(-5+i) & 1 & 0 \\ -2 & -4-(-5+i) & 0 \end{array}\right) \Rightarrow \left(\begin{array}{cc|c} -1-i & 1 & 0 \\ -2 & 1-i & 0 \end{array}\right)$

$\left(\begin{array}{cc|c} 1 & -\frac{(1-i)}{2} & 0 \\ 0 & 0 & 0 \end{array}\right)$ $\quad x - \dfrac{(1-i)}{2} y = 0$

$K_1 \begin{pmatrix} 1-i \\ 2 \end{pmatrix}$

$B_1 = \operatorname{Re} K_1 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$, $B_2 = \operatorname{Im} K_1 = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$

$X_1 = [B_1 \cos\beta t - B_2 \sin\beta t] e^{\alpha t}$, $\alpha = -5$, $\beta = 1$

$= \left[\begin{pmatrix} 1 \\ 2 \end{pmatrix} \cos t - \begin{pmatrix} -1 \\ 0 \end{pmatrix} \sin t\right] e^{-5t}$

$X_2 = [B_2 \cos\beta t + B_1 \sin\beta t] e^{\alpha t}$

$= \left[\begin{pmatrix} -1 \\ 0 \end{pmatrix} \cos t + \begin{pmatrix} 1 \\ 2 \end{pmatrix} \sin t\right] e^{-5t}$

$\Rightarrow \boxed{X = c_1 \begin{pmatrix} \cos t + \sin t \\ 2\cos t \end{pmatrix} e^{-5t} + c_2 \begin{pmatrix} -\cos t + \sin t \\ 2\sin t \end{pmatrix} e^{-5t}}$

$t \in (-\infty, \infty)$.