MATH 202.10 (Term 151)

Quiz 5 (Sects. 6.2 & 6.3) **Duration: 20min**

Name: **ID number:**

1.)(7pts)Find first four nonzero terms of 2 powers series solutions of: $(x^2-4x)y''-y'-2y=0$.

2.)(3pts)Find the indicial roots of: $x^2y''+2x(x^2-x+1)y'-(1-x)y=0$ **at** $x_0=0$.

1) $x=0$ is a regular singular point of the DE

There exists at least one solution

$$y=\sum_{n=0}^{\infty} c_n x^{n+r}, \quad 0<x<R.$$

$$(x^2-4x)\sum_{n=0}^{\infty} c_n(n+r)(n+r-1)x^{n+r-2}-\sum_{n=0}^{\infty} c_n(n+r)x^{n+r-1}-2\sum_{n=0}^{\infty} c_n x^{n+r}=0$$

$$x^r\left[\sum_{n=0}^{\infty} c_n(n+r)(n+r-1)x^n-4\sum_{n=0}^{\infty} c_n(n+r)(n+r-1)x^{n-1}-\sum_{n=0}^{\infty} c_n(n+r)x^{n-1}-2\sum_{n=0}^{\infty} c_n x^n\right]=0.$$

$$-4c_0r(r-1)x^{-1}-c_0rx^{-1}+\sum_{k=0}^{\infty} c_k(k+r)(k+r-1)x^k-4\sum_{k=0}^{\infty} c_{k+1}(k+1+r)(k+r)x^k-\sum_{k=0}^{\infty} c_{k+1}(k+1+r)x^k-2\sum_{k=0}^{\infty} c_k x^k=0$$

$$-c_0r(4r-3)x^{-1}+\sum_{k=0}^{\infty}\left[c_k[(k+r)(k+r-1)-2]+c_{k+1}(k+r+1)(-4k-4r-1)\right]x^k=0$$

$$\begin{cases} c_0r(4r-3)=0 \\ c_{k+1}=\dfrac{(k+r)(k+r-1)-2}{(k+r+1)(4r+4k+1)}c_k, \quad k=0,1,2, \end{cases}$$

$r=0$, $r=3/4$

Case 1 $r=0$

$$c_{k+1}=\frac{k(k-1)-2}{(k+1)(4k+1)}c_k$$

$c_1=-2c_0$, $c_2=\frac{-2}{10}c_1=\frac{2}{5}c_0$

$$y=c_0+c_1x+c_2x^2+\cdots$$
$$=c_0\underbrace{\left(1-2x+\tfrac{2}{5}x^2+\cdots\right)}_{y_1}$$

Case 2: $r=3/4$

$$c_{k+1}=\frac{(k+\frac{3}{4})(k-\frac{1}{4})-2}{(4k+7)(k+1)}c_k,$$

$c_1=\frac{-35}{16\cdot 7}c_0$, $c_2=\frac{11/16}{22}c_1=$

$$y=x^{3/4}(c_0+c_1x+c_2x^2+\cdots)$$
$$=c_0\underbrace{x^{3/4}\left(1-\frac{35}{16\cdot 7}x+\cdots\right)}_{y_2}$$

2) $y''+\underbrace{\frac{2}{x}(x^2-x+1)}_{P}y'-\underbrace{\frac{(1-x)}{x^2}}_{Q}y=0$

$p(x)=xP(x)=2(x^2-x+1)$

$q(x)=x^2Q=-(1-x)$

$r(r-1)+2r-1=0$

$r^2+r-1=0$

$\Delta=1+4=5$

$r_1=\frac{-1-\sqrt{5}}{2}$

$r_2=\frac{-1+\sqrt{5}}{2}$