MATH 202.10 (Term 151)

Quiz 3 (Sects 4.2 & 4.3)

Duration: 20 mins

Name:

ID:

1.) (5 pts) Use reduction of order to find a second solution of the DE $(1+x)y''+xy'-y=0$ given that $y_1=e^{-x}$ is a solution.

2.) (5 pts) Solve the IVP $\begin{cases} y'''+y''+2y'+2y=0 \\ y(0)=0,\ y'(0)=1,\ y''(0)=0 \end{cases}$

1.) $y''+\frac{x}{1+x}y'-\frac{1}{1+x}y=0$

$P(x)=\frac{x}{1+x}$

$y_2=y_1\int\frac{e^{-\int P(x)\,dx}}{y_1^2}\,dx$

$=e^{-x}\int\frac{e^{-\int\frac{x}{1+x}dx}}{e^{-2x}}\,dx$

$-\int\frac{x}{1+x}dx=-\int\left(1-\frac{1}{1+x}\right)dx$

$=-x+\ln(1+x),\ x>-1$

$\Rightarrow y_2=e^{-x}\int\frac{(x+1)e^{-x}}{e^{-2x}}\,dx$

$=e^{-x}\int(1+x)e^{x}\,dx$

$=e^{-x}\,xe^{x}$

$\boxed{y_2=x}$

2.) The auxilliary equation is

$m^3+m^2+2m+2=0$

$(m+1)(m^2+2)=0$

$m=-1,\quad m=\pm\sqrt{2}\,i$

$y=c_1e^{-x}+c_2\cos\sqrt{2}x+c_3\sin\sqrt{2}x$

Now,

$y'=-c_1e^{-x}-\sqrt{2}c_2\sin\sqrt{2}x+\sqrt{2}c_3\cos\sqrt{2}x$

$y''=c_1e^{-x}-2c_2\cos\sqrt{2}x-2c_3\sin\sqrt{2}x$

$y(0)=0 \Rightarrow \begin{cases} c_1+c_2=0 & (1) \\ -c_1+\sqrt{2}c_3=1 & (2) \\ c_1-2c_2=0 & (3) \end{cases}$

$y'(0)=1 \Rightarrow$

$y''(0)=0 \Rightarrow$

(1) and (3) $\Rightarrow c_2=c_1=0$

(2) $\Rightarrow c_3=\frac{1}{\sqrt{2}}=\frac{\sqrt{2}}{2}$

$\Rightarrow \boxed{y=\frac{\sqrt{2}}{2}\sin\sqrt{2}x}$