MATH 202.10 (term 151)

Quiz 2 (Sects 2.4 & 2.5) Duration: 20 min

Name:

ID:

1.) (5 pts) Solve the exact DE $(ye^{-2xy} + \sin^2 x)dx + xe^{-2xy}dy = 0$

2.) (5 pts) Solve by substitution the DE $x^2 \frac{dy}{dx} + y = y^{1/4}$

---

1) $M = ye^{-2xy} + \sin^2 x$, $N = xe^{-2xy}$

$M_y = e^{-2xy} - 2xye^{-2xy}$

$N_x = e^{-2xy} - 2xye^{-2xy}$

$M_y = N_x \Rightarrow$ Exact DE

$$\begin{cases} \frac{\partial f}{\partial x} = ye^{-2xy} + \sin^2 x & (1) \\ \frac{\partial f}{\partial y} = xe^{-2xy} & (2) \end{cases}$$

We integrate (2), we find

$f(x,y) = -\frac{1}{2}e^{-2xy} + g(x)$

We substitute into (1).

$ye^{-2xy} + g'(x) = ye^{-2xy} + \sin^2 x$

$g'(x) = \sin^2 x$

$g(x) = \int \sin^2 x \, dx$

$= \frac{1}{2}\int(1 - \cos 2x)\,dx$

$= \frac{1}{2}\left(x - \frac{\sin 2x}{2}\right)$

$-\frac{1}{2}e^{-2xy} + \frac{1}{2}\left(x - \frac{\sin 2x}{2}\right) = C$

$$\boxed{-e^{-2xy} + x - \frac{\sin 2x}{2} = C}$$

2) This is Bernouilli's DE

$u = y^{1-\frac{1}{4}} = y^{\frac{3}{4}}$, $y = u^{\frac{4}{3}}$

$\frac{du}{dx} = \frac{3}{4}y^{-\frac{1}{4}}\frac{dy}{dx}$, $\frac{dy}{dx} = \frac{4}{3}u^{\frac{1}{3}}\frac{du}{dx}$

We substitute into the DE

$\frac{4}{3}x^2 u^{\frac{1}{3}}\frac{du}{dx} + u^{\frac{4}{3}} = u^{\frac{1}{3}}$

$$\boxed{\frac{4}{3}x^2\frac{du}{dx} + u = 1}$$ linear DE

$\frac{du}{dx} + \frac{3}{4x^2}u = \frac{3}{4x^2}$, $x > 0$

An integrating factor is

$e^{\int \frac{3}{4x^2}dx} = e^{-\frac{3}{4x}}$

$\Rightarrow \frac{d}{dx}\left(ue^{-\frac{3}{4x}}\right) = \frac{3}{4x^2}e^{-\frac{3}{4x}}$

$ue^{-\frac{3}{4x}} = \frac{3}{4}\int \frac{1}{x^2}e^{-\frac{3}{4x}}dx$

$= e^{-\frac{3}{4x}} + C$

$u = 1 + Ce^{\frac{3}{4x}}$

$y^{\frac{3}{4}} = 1 + Ce^{\frac{3}{4x}}$

$$\boxed{y = \left(1 + Ce^{\frac{3}{4x}}\right)^{\frac{4}{3}}, \quad x > 0}$$