MATH 202.10 (151)

Quiz 1 (Sects 1.2, 2.2, 2.3) Duration: 20 min

Name:

ID:

1.) (2 pts) Do the following IVP have unique solutions?

a.) $\begin{cases} (x-1)\dfrac{dy}{dx} - \sqrt{y+1} = 0 \\ y(0) = -1 \end{cases}$ b.) $\begin{cases} x^2\dfrac{dy}{dx} + (\ln x)y = 1 \\ y(1) = 1 \end{cases}$

2.) (4 pts) Solve the ODE: $y\,dy = \cos^3 x \sin x\,(y^2-y-2)\,dx$

3.) (4 pts) Solve the ODE: $(x+1)^3 dy - [1-(x+1)^2 y]dx = 0$

1.) $\dfrac{dy}{dx} = \dfrac{\sqrt{y+1}}{x-1}$, $f(x,y) = \dfrac{\sqrt{y+1}}{x-1}$

a) $\dfrac{\partial f}{\partial y}(x,y) = \dfrac{1}{2(x-1)\sqrt{y+1}}$

$\dfrac{\partial f}{\partial y}$ is not continuous at $(0,-1)$

The IVP may have one or many solutions

b) $\dfrac{dy}{dx} = \dfrac{1-(\ln x)y}{x^2} = f(x,y)$

$\dfrac{\partial f}{\partial y} = -\dfrac{\ln x}{x^2}$

f and $\dfrac{\partial f}{\partial y}$ are continuous at $(1,1)$

The IVP has a unique solution.

2.) $\displaystyle\int \frac{y}{y^2-y-2}\,dy = \int \cos^3 x \sin x\,dx$

$\dfrac{y}{y^2-y-2} = \dfrac{1/3}{y+1} + \dfrac{2/3}{y-2}$

$\dfrac{1}{3}\ln|y+1| + \dfrac{2}{3}\ln|y-2| = -\dfrac{\cos^4 x}{4} + C$

$\ln|(y+1)(y-2)^2| = -\dfrac{3}{4}\cos^4 x + C$

$\boxed{(y+1)(y-2)^2 = C\,e^{-\frac{3}{4}\cos^4 x}}$

is an explicit solution

3.) $(x+1)^3\dfrac{dy}{dx} + (x+1)^2 y = 1$

$\dfrac{dy}{dx} + \dfrac{1}{x+1}y = \dfrac{1}{(x+1)^3}$

An integrating factor is

$e^{\int \frac{1}{x+1}dx} = e^{\ln|x+1|} = x+1, \quad x > -1$

$\Rightarrow \dfrac{d}{dx}\big(y(x+1)\big) = \dfrac{1}{(x+1)^2}$

$y(x+1) = \displaystyle\int \frac{1}{(x+1)^2}\,dx + C$

$= -\dfrac{1}{x+1} + C$

$\boxed{y = \dfrac{1}{x+1}\left(-\dfrac{1}{x+1} + C\right), \quad x \in (-1,\infty)}$