KFUPM, DEPARTMENT OF MATHEMATICS AND STATISTICS

MATH 202 : TEST 2, SEMESTER (151), OCTOBER 11, 2015

Name : ....

ID : ....

Exercise	**Points**
1	: **10**
2	: **6**
3	: **12**
4	: **6**
5	: **6**
6	: **6**
7	: **6**
Total	: **52**

Exercise 1. Solve the IVP :

$$\left[(x^5 + y^5)\ \mathrm{dx} + xy^4\ \mathrm{dy} = 0, \quad y(1) = 2\right]$$

Exercise 2. Use an appropriate substitution to transform the following DE :

$$y' + \frac{3y}{7x^2 + 11} = \frac{2x}{(7x^2 + 11)\sqrt{y}}$$

to a linear DE.

Exercise 3. Consider the DE :

$$(3y + 3y^3)\ \mathrm{dx} + (xy^2 - x)\ \mathrm{dy} = 0$$

(i) Is the given DE exact ?
(ii) Find all $\alpha, \beta \in \mathbb{R}$ such that $\mu(x, y) = x^\alpha y^\beta$ is an integrating factor of the DE.
(iii) Solve the DE.

Exercise 4. A thermometer reading $15°C$ is brought into a room where the temperature is $30°C$, 2 minutes later the thermometer reads $20°C$. Find the thermometer reading 3 minutes after it was brought into the room.

Exercise 5. Consider the functions defined on the interval $I = \mathbb{R}$ by : $f_1(x) = x^2$, $f_2(x) = xe^x$.

(i) Show that f_1, f_2 are linearly independent.

(ii) Is there a homogeneous second order linear DE with $\{f_1, f_2\}$ as a fundamental set of solutions ?

Exercise 6. Find an interval centered at 0 for which the following IVP :

$$(x-1)y'' + (\sec x)y' + \frac{1}{x+1}y = x^2, \quad y(0) = 1, y'(0) = 1$$

has a unique solution.

Exercise 7. Suppose that $y = c_1x+c_2xe^x+1$ is a two-parameters family of solutions of a DE. Determine whether a member of the family can be found that satisfies the boundary conditions $y(1) = 0$, $y(-1) = 3$.