King Fahd University of Petroleum and Minerals

Department of Mathematics and Statistics

Math 202
Exam I – 2015–2016 (151)
Monday, October 12, 2015

Allowed Time: 2 Hours

Name: ____

ID #: _____________________________

Section #: ________________ Serial Number: ________________

Instructions:

1. Write clearly and legibly. You may lose points for messy work.
2. **Show all your work**. No points for answers without justifications.
3. **Calculators and Mobiles are not allowed.**
4. Make sure that you have **9 different** problems (9 pages + cover page)

Question #	Grade	Maximum Points
1		08
2		10
3		10
4		12
5		21
6		12
7		10
8		10
9		07
Total:		100

1. (a) Given that $y = \frac{3(x+c)-1}{x+c}$, c a real constant, is a one parameter family of solutions of the differential equation:

$$\frac{dy}{dx} = (y-3)^2\ ,$$

find a singular solution for this differential equation. (**Justify your answer**).

(b) Find the values of m such that $y(x) = m\,x\,\ln\,x,\ \ x > 0$, is a solution of the differential equation:

$$x^2\,(y'')^2 + y' - \frac{y}{x} = 0,\ \ x\ > 0.$$

2. (a) Find a region R in the $xy-$plane on which the following initial value problem (IVP):

$$y' = \sqrt{y^2 - 25} + \sqrt{4 - x^2} + \frac{1}{x}, \quad y(x_0) = y_0, \tag{1}$$

has a unique solution for every $(x_0, y_0) \in R$. Sketch R.

(b) Find the largest interval on which the solution of the IVP (1) may be defined, for $x_0 = 1,\ y_0 = 6$.

3. Solve the initial value problem:

$$\begin{cases} (t - t^2)x' = \sqrt{1 - x^2} \\ x\left(\dfrac{1}{2}\right) = 1\,. \end{cases}$$

4. Solve the differential equation:

$$ty' + y - t^4 \ln t = 0,$$

and find its interval of solution.

5. (a) Solve the differential equation:

$$(3x^5 + 2x^3y^2 - 20x^3)dx + x^4y\,dy = 0.$$

(b) Consider the differential equation:

$$[4 - 4(x+1)\cos^2 y]dx + (x+1)^2 \sin 2y\,dy = 0. \qquad (2)$$

(i) Determine whether the differential equation (2) is Exact or not.

(ii) In case that the differential equation (2) is not Exact, find an integrating factor which makes equation (2) Exact. **(Do not solve the obtained equation !)**

6. (a) Find a suitable substitution that transforms the differential equation

$$y^{\frac{1}{3}}(x^2+1)(x-1)^{\frac{4}{3}}\,\frac{dy}{dx}+(x^2+1)(x-1)^{\frac{1}{3}}\,y^{\frac{4}{3}}=1,$$

into a **linear** differential equation. Find the new linear equation but **do not solve it**.

(b) Find a suitable substitution that transforms the differential equation

$$(x^2+2\,y^2)y'=xy$$

into a **separable** differential equation. Find the new separable equation but **do not solve it**.

7. A body of temperature 100°C is put in a room of temperature 25°C. If it takes the body 15 minutes to cool down to 70°C, then after how much time the temperature of the body reaches 40° C .

8. (a) Verify that $x = c_1 \cos 5t + c_2 \sin 5t$ is a two-parameter family of solutions of the differential equation:

$$x'' + 25\,x = 0. \qquad (3)$$

(b) Determine whether a member of the family of solutions of the differential equation (3) can be found that satisfies the boundary conditions:

i)- $$x(0) = 0, \quad x(\tfrac{\pi}{5}) = 1.$$

ii)- $$x(\tfrac{\pi}{5}) = 7, \quad x'(\tfrac{\pi}{5}) = 11.$$

9. Given that $y_1(x) = 1$ and $y_2(x) = \cos(2x)$ are solutions for the homogeneous differential equation:

$$y'' + b(x)\,y' + c(x)\,y = 0$$

where $b(x)$, $c(x)$ are two polynomials. Show that $y(x) = \sin^2 x$ is also a solution for the homogeneous differential equation.