

**KING FAHD UNIVERSITY OF PETROLEUM AND MINERALS**  
**DEPARTMENT OF MATHEMATICS AND STATISTICS**  
**MATH 131 FINAL EXAM**

**Duration: 150 mn**

**Instructor: Dr. Bilal Chanane**

NAME:.....ID:.....

**Note: 50 pts for the exercise on the use of the simplex method, all the other exercises have a 25 pts weight. The total is 400 pts.**

**Exercise 1** *If  $X$  is a normal random variable with mean  $\mu = 30$  and standard deviation  $\sigma = 5$ , determine the value of the standard normal random variable  $Z$  that corresponds to  $X = 28$ .*

**Exercise 2** *If  $Z$  has a standard normal distribution, find  $P(-1 < Z < 2)$ . Hint:  $A(1.) = 0.3413$ ,  $A(2.) = 0.4772$*

**Exercise 3** *The time (in minutes) that a person arriving at a train station must wait for a train is uniformly distributed with density  $f(x) = \frac{1}{20}$  where  $0 \leq x \leq 20$ . Find the mean waiting time  $\mu$  and the standard deviation  $\sigma$ .*

**Exercise 4** *Suppose the cumulative distribution function of the random variable  $X$  is given by*

$$F(x) = \begin{cases} 0, & \text{if } x < 0 \\ \frac{x}{4}, & \text{if } 0 \leq x \leq 4 \\ 1, & \text{if } x > 4 \end{cases}$$

*Find  $P(1 < X < 3)$ .*

**Exercise 5** *A biased coin is tossed 7 times. If the probability of heads appearing on any toss is  $\frac{2}{3}$ , what is the probability that exactly three heads appear.*

**Exercise 6** *A random variable  $X$  has a distribution given by  $f(0) = 0.3$ ,  $f(1) = k$ ,  $f(2) = 0.5$ . Find the mean  $\mu$  and the variance  $\text{Var}(X)$ ,*

**Exercise 7** *A sample space is partitioned by events  $E$  and  $F$ , where  $P(E) = \frac{1}{2}$ . Suppose  $S$  is an event such that  $P(S|E) = \frac{1}{3}$  and  $P(S|F) = \frac{3}{5}$ . Find  $P(F|S)$ .*

**Exercise 8** *The probability that Bob survives ten more years is  $\frac{3}{5}$ , and the probability that Mary survives ten more years is  $\frac{2}{3}$ . Find the probability that exactly one of them survives ten more years. (Assume independence). Hint: Draw a diagram..*

**Exercise 9** *After a production run, it was found that 20% of the units produced had a faulty weld and 15% had both a defective paint job and a faulty weld. If a unit is randomly selected from this run and it has a faulty weld, what is the probability that it also has a defective paint job?*

**Exercise 10** *If a pair of dice are rolled, what is the probability that the sum of the numbers appearing is 10?*

**Exercise 11** *if a coin is tossed and then a die is rolled, and the results are observed, determine the sample space of this experiment.*

**Exercise 12** An \$10000 loan is amortized by equal semiannual payments over five years. If interest is at the rate of 8% compounded semiannually, what is the first payment ?

**Exercise 13** Determine the present value of \$7000 due in 4 years if the interest rate is 7% compounded quarterly?

**Exercise 14** Use the simplex method to solve the linear programming problem

$$\text{Maximize } Z = 20x + 30y$$

subject to

$$3x + y \leq 10$$

$$x + 8y \leq 11$$

$$x + 3y \leq 15$$

$$x, y \geq 0$$

**Exercise 15** Find the dual of

$$\text{Maximize } U = 2x + y + 4z$$

subject to

$$x + 2y - z \leq 4$$

$$4x - y + z \leq 9$$

$$x + 3y - z \leq 5$$

$$x, y, z \geq 0$$