KING FAHD UNIVERSITY OF PETROLEUM AND MINERALS DEPARTMENT OF MATHEMATICS AND STATISTICS MATH 131 FINAL EXAM

Duration: 150 mn

Instructor: Dr. Bilal Chanane

NAME:ID:

Note: 50 pts for the exercise on the use of the simplex method, all the other exercises have a 25 pts weight. The total is 400 pts.

Exercise 1 If X is a normal random variable with mean $\mu = 16$ and standard deviation $\sigma = 2$, determine the value of the standard normal random variable Z that corresponds to X = 23.

Exercise 2 If Z has a standard normal distribution, find P(-0.65 < Z < 1.92). Hint: A(0.65) = 0.2422, A(1.92) = 0.4726

Exercise 3 The time (in minutes) that a person arriving at a bus stop must wait for a bus is uniformly distributed with density $f(x) = \frac{1}{15}$ where $0 \le x \le 15$. Find the mean waiting time μ and the standard deviation σ .

Exercise 4 Suppose the cumulative distribution function of the random variable X is given by

$$F(x) = \begin{cases} 0, & \text{if } x < 0\\ \frac{x^2}{25}, & \text{if } 0 \le x \le 5\\ 1, & \text{if } x > 5 \end{cases}$$

Find P(2 < X < 3).

Exercise 5 A biased coin is tossed 8 times. If the probability of heads appearing on any toss is $\frac{1}{3}$, what is the probability that exactly six heads appear.

Exercise 6 A random variable X has a distribution given by f(0) = 0.3, f(1) = 0.6, f(2) = k. Find the mean μ and the standard deviation σ .

Exercise 7 A sample space is partitioned by events E and F, where $P(E) = \frac{1}{3}$. Suppose S is an event such that $P(S|E) = \frac{1}{4}$ and $P(S|F) = \frac{4}{5}$. Find P(F|S).

Exercise 8 The probability that Bob survives ten more years is $\frac{4}{5}$, and the probability that Mary survives ten more years is $\frac{5}{6}$. Find the probability that exactly one of them survives ten more years. (Assume independence). Hint: Draw a diagram.

Exercise 9 After a production run, it was found that 10% of the units produced had a faulty weld and 5% had both a defective paint job and a faulty weld. If a unit is randomly selected from this run and it has a faulty weld, what is the probability that it also has a defective paint job?

Exercise 10 If a pair of dice are rolled, what is the probability that the sum of the numbers appearing is 5?

Exercise 11 if a die is rolled and then a coin is tossed, and the results are observed, determine the sample space of this experiment.

Exercise 12 An \$800 loan is amortized by equal quaterly payments over two years. If interest is at the rate of 16% compounded quarterly, what is the quaterly payment?

Exercise 13 Determine the present value of \$4000 due in 5 years if the interest rate is 10% compounded semiannually.

Exercise 14 Use the simplex method to solve the linear programming problem

$$Maximize\ Z = 30x + 50y$$

subject to

$$\begin{array}{rcl} 2x+y & \leq & 16 \\ x+2y & \leq & 11 \\ x+3y & \leq & 15 \\ x,y & \geq & 0 \end{array}$$

Exercise 15 Find the dual of

$$Maximize \ U = x + 2y + 4z$$

$$subject\ to$$

$$3x + y - z \leq 4$$

$$4x - y + 2z \leq 9$$

$$x + 3y - z \leq 15$$

$$x, y, z \geq 0$$