King Fahd University of Petroleum and Minerals Name:..
Department of Mathematics and Statistics ID:..
Math 131, Instructor: Dr. Bilal Chanane Section: ....
Major Exam 2, Duration 1:30 h Grade:..

DO NOT WRITE ON THIS SHEET
A good presentation is a must
justify all your answers

Exercise 1: Use the dual and the simplex method to solve the linear programming problem

$$\text{Minimize } Z = 3x + 2y$$

subject to

$$\begin{cases} 5x + 3y \geq 25 \\ x + 2y \geq 7 \\ x, \ y \geq 0 \end{cases}$$

Exercise 2: Find the dual (do not solve) of

$$\text{Maximize } Z = 2x_1 + 6x_2 + 5x_3$$

subject to

$$\begin{cases} x_1 + 2x_2 + 3x_3 \leq 15 \\ 3x_1 + 4x_2 + 7x_3 \leq 4 \\ 4x_1 + x_2 + x_3 \leq 20 \\ 5x_1 + 2x_2 + 4x_3 \leq 3 \\ x_1, \ x_2, \ x_3 \geq 0 \end{cases}$$

Exercise 3: Use the simplex method to solve

$$\text{Maximize } Z = 2x_1 + 5x_2 + 4x_3$$

subject to

$$\begin{cases} x_1 + 2x_2 + 3x_3 \leq 15 \\ 3x_1 + 2x_2 + x_3 \leq 14 \\ x_1, \ x_2, \ x_3 \geq 0 \end{cases}$$

Exercise 4: Use the graphical method to solve the linear programing problem

$$\text{Maximize } Z = 8x_1 + 2x_2$$

subject to

$$\begin{cases} 2x_1 + 2x_2 \leq 40 \\ 11x_1 + x_2 \leq 49 \\ 4x_1 + x_2 \leq 25 \\ x_1, \ x_2 \geq 0 \end{cases}$$

Exercise 5: An oil company that has two refineries needs at least 8000, 14000 and 5000 barrels of low-, medium-, and high-grade oil, respectively. Each day, Refinery I produces 2000 barrels of low-, 3000 barrels of medium-, and 1000 barrels of high-grade oil, whereas Refinery II produces 1000 barrels each of low-, and high-, and 2000 barrels of medium-grade oil. If it costs $25000 per day to operate Refinery I and $20000 per day to operate Refinery II, how many days should each refinery be operated to satisfy the production requirements at minimum cost? What is the minimum cost? (Assume a minimum cost exists).