MATH 102.5 (Term 151)

Quiz 4 (Sects. 8.2, 8.3 & 8.4) Duration: 30mn

Name: ID number:

1.)(4pts)Evaluate the integral $I = \int \frac{1}{x^2\sqrt{9-x^2}}dx$.

2.)(4pts)Evaluate the integral $J = \int \frac{x^2+1}{(x^2+x-2)x}dx$.

3.)(2pts)Write the function $f(x) = \frac{2x^4+1}{(x^2+1)(x^2+x+1)^2(x+1)}$ in partial fraction (Do not evaluate the constants).

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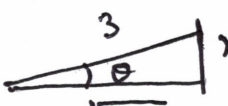
1) $I = \int \frac{1}{x^2\sqrt{9-x^2}}dx$

$x = 3\sin\theta \quad , \quad -\frac{\pi}{2} < \theta < \frac{\pi}{2}$

$dx = 3\cos\theta\, d\theta$

$\Rightarrow I = \int \frac{3\cos\theta\, d\theta}{9\sin^2\theta\,(3\cos\theta)} = \frac{1}{9}\int \frac{d\theta}{\sin^2\theta}$

$I = -\frac{1}{9}\cot\theta + C$




$\cos\theta = \frac{\sqrt{9-x^2}}{3}$

$\Rightarrow \cot\theta = \frac{\cos\theta}{\sin\theta} = \frac{\sqrt{9-x^2}}{x}$

$\Rightarrow \boxed{I = -\frac{\sqrt{9-x^2}}{x} + C}$

2) $J = \int \frac{x^2+1}{(x^2+x-2)x}dx$

$\frac{x^2+1}{(x^2+x-2)x} = \frac{a}{x} + \frac{b}{x+2} + \frac{c}{x-1}$

$\frac{x^2+1}{x^2+x-2} = a + x\left(\frac{b}{x+2} + \frac{c}{x-1}\right)$

$x=0 \Rightarrow \boxed{a = -1/2}$

$\frac{x^2+1}{x(x-1)} = b + (x+2)\left(\frac{a}{x} + \frac{c}{x-1}\right)$

$x=-2 \Rightarrow \boxed{b = \frac{5}{6}}$

$\frac{x^2+1}{x(x+2)} = c + (x-1)\left(\frac{a}{x} + \frac{b}{x+2}\right)$

$x=1 \Rightarrow \boxed{c = \frac{2}{3}}$

$J = \int \left(\frac{-1/2}{x} + \frac{5/6}{x+2} + \frac{2/3}{x-1}\right)dx$

$= -\frac{1}{2}\ln|x| + \frac{5}{6}\ln|x+2| + \frac{2}{3}\ln|x-1| + C$

3) $f(x) = \frac{a}{x+1} + \frac{bx+c}{x^2+1} + \frac{dx+e}{x^2+x+1} + \frac{fx+h}{(x^2+x+1)^2}$