MATH 102.5 (Term 151)

Quiz 3 (Sects. 7.1, 7.3 & 8.1) Duration: 20min

Name: ID number:

1.)(3pts)Let $y = \log_2(\frac{1}{x})$. Find $y'(e)$.
2.)(3pts)Find a real number a such that $\sinh(2a) = -1$.
3.)(4pts)Evaluate $I = \int_1^e \sin(3\ln x)dx$.

1.) $y = \log_2\left(\frac{1}{x}\right) = \frac{\ln(1/x)}{\ln 2} = -\frac{\ln x}{\ln 2}$

$y' = -\frac{1}{x\ln 2} \Rightarrow \boxed{y'(e) = -\frac{1}{e\ln 2}}$

2.) $\sinh 2a = -1$

$\Leftrightarrow \frac{e^{2a} - e^{-2a}}{2} = -1$

$\Leftrightarrow e^{2a} + 2 - e^{-2a} = 0$

$\Leftrightarrow e^{4a} + 2e^{2a} - 1 = 0$

let $x = e^{2a}$

$x^2 + 2x - 1 = 0$

$\Delta = 4 + 4 = 8$

$x_1 = \frac{-2 - 2\sqrt{2}}{2} = -1 - \sqrt{2}$

$x_2 = \frac{-2 + 2\sqrt{2}}{2} = -1 + \sqrt{2}$

x_1 rejected, must be positive

$e^{2a} = -1 + \sqrt{2}$

$2a = \ln(-1 + \sqrt{2})$

$\boxed{a = \frac{1}{2}\ln(-1+\sqrt{2})}$

3.) $I = \int_1^e \sin(3\ln x)dx$

$u' = 1 \longrightarrow u = x$

$v = \sin(3\ln x) \longrightarrow v' = \frac{3}{x}\cos(3\ln x)$

$I = \left[x\sin(3\ln x)\right]_1^e - \int_1^e 3\cos(3\ln x)dx$

$u' = 1 \longrightarrow u = x$

$v = \cos(3\ln x) \longrightarrow v' = -\frac{3}{x}\sin(3\ln x)$

Thus,

$I = \left[x\sin(3\ln x)\right]_1^e - 3\left[\left[x\cos(3\ln x)\right]_1^e + 3\int_1^e \sin(3\ln x)dx\right]$

$= \left[x\sin(3\ln x) - 3x\cos(3\ln x)\right]_1^e - 9I$

$10I = e\sin(3) - 3e\cos 3 + 3$

$\boxed{I = \frac{e\sin 3 - 3e\cos 3 + 3}{10}}$