King Fahd University of Petroleum and Minerals
Department of Mathematics

Math 102
Exam I
Term 151
Tuesday 13/10/2015
Net Time Allowed: 120 minutes

KEY
+ Detailed Solutions

MASTER VERSION

Q	MM	V1	V2	V3	V4
1	a	b	a	b	e
2	a	c	b	d	b
3	a	d	b	b	d
4	a	c	b	b	d
5	a	d	b	b	d
6	a	e	e	a	b
7	a	b	e	e	a
8	a	a	b	d	d
9	a	a	e	d	a
10	a	d	d	d	b
11	a	b	b	c	d
12	a	d	e	e	b
13	a	e	e	b	a
14	a	b	e	e	e
15	a	e	e	c	a
16	a	d	a	b	e
17	a	e	a	c	a
18	a	a	d	a	e
19	a	c	a	d	a
20	a	c	d	c	a

1. If P is a partition of $[0,2]$, then $\lim_{\|P\|\to 0} \sum_{k=1}^{n} \frac{1}{c_k+2} \triangle x_k =$

$\int_0^2 \frac{1}{x+2}\,dx = \ln|x+2|\Big]_0^2$

$= \ln 4 - \ln 2$

$= \ln\left(\frac{4}{2}\right)$

$= \ln 2$

(a) $\ln 2$

(b) $\ln 4$

(c) $\frac{1}{4}$

(d) $2 - \ln 2$

(e) 2

2. $\int \frac{(1-x)(1-2x)}{x^2}\,dx =$ $\int \frac{1-3x+2x^2}{x^2}\,dx = \int \frac{1}{x^2} - \frac{3}{x} + 2\,dx$

$= -\frac{1}{x} - 3\ln|x| + 2x + C$

(a) $2x - \frac{1}{x} - 3\ln|x| + C$

(b) $2x + \frac{1}{x} - 4\ln|x| + C$

(c) $-2x - \frac{1}{x} + 3\ln|x| + C$

(d) $2 - \frac{1}{x} - \frac{2}{x^2} + C$

(e) $x - \frac{3}{2}x^2 + x^3 + C$

3. $\int (\theta - 1)\cos\left(\frac{\theta^2}{2} - \theta + 1\right) d\theta =$

$u = \frac{\theta^2}{2} - \theta + 1 \Rightarrow du = (\theta - 1)\, d\theta$

$= \int \cos u \, du$

$= \sin u + C$

$= \sin\left(\frac{\theta^2}{2} - \theta + 1\right) + C$

(a) $\sin\left(\frac{\theta^2}{2} - \theta + 1\right) + C$

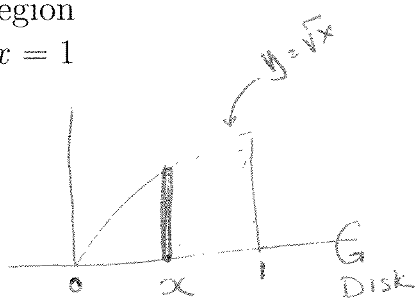
(b) $(\theta - 1)\sin\left(\frac{\theta^2}{2} - \theta + 1\right) + C$

(c) $\cos\left(\frac{\theta^2}{2} - \theta + 1\right) + C$

(d) $\frac{1}{2}(\theta - 1)^2 \cos\left(\frac{\theta^2}{2} - \theta + 1\right) + C$

(e) $\cos^2\left(\frac{\theta^2}{2} - \theta + 1\right) + C$

4. The **volume** of the solid generated by rotating the region bounded the curves $y = \sqrt{x}$ and $y = 0$ from $x = 0$ to $x = 1$ about the x-axis is equal to



(a) $\frac{\pi}{2}$

(b) $\frac{\pi}{4}$

(c) π

(d) 1

(e) 2π

$V = \int_0^1 \pi (\sqrt{x})^2 \, dx$

$= \pi \int_0^1 x \, dx$

$= \pi \cdot \frac{1}{2} x^2 \Big]_0^1$

$= \frac{\pi}{2}$

5. $\int_0^2 |x^2-2|\,dx =$

(a) $\dfrac{-4+8\sqrt{2}}{3}$

(b) $\dfrac{-4+4\sqrt{2}}{3}$

(c) $\dfrac{2-3\sqrt{2}}{3}$

(d) $\dfrac{4+\sqrt{2}}{3}$

(e) $\dfrac{2+4\sqrt{2}}{3}$

$x^2-2=0 \implies x=\pm\sqrt{2}$

$+ \quad -\sqrt{2} \quad - \quad \sqrt{2} \quad +$

$$= \int_0^{\sqrt{2}} |x^2-2|\,dx + \int_{\sqrt{2}}^{2} |x^2-2|\,dx$$

$$= \int_0^{\sqrt{2}} 2-x^2\,dx + \int_{\sqrt{2}}^{2} x^2-2\,dx$$

$$= 2x-\tfrac{1}{3}x^3\Big]_0^{\sqrt{2}} + \tfrac{1}{3}x^3-2x\Big]_{\sqrt{2}}^{2}$$

$$= \left(2\sqrt{2}-\tfrac{2}{3}\sqrt{2}\right) + \left(\tfrac{8}{3}-4\right)-\left(\tfrac{2\sqrt{2}}{3}-2\sqrt{2}\right)$$

$$= \frac{4\sqrt{2}}{3} + \left(-\frac{4}{3}\right) - \left(-\frac{4}{3}\sqrt{2}\right)$$

$$= \frac{8\sqrt{2}}{3}-\frac{4}{3} = \frac{-4+8\sqrt{2}}{3}$$

6. $\int_0^{\sqrt{3}} \sqrt{12-4x^2}\,dx =$

[Hint: Interpret the integral in terms of area]

(a) $\dfrac{3\pi}{2}$

(b) $\dfrac{\pi\sqrt{3}}{2}$

(c) $2\pi\sqrt{3}$

(d) $\dfrac{3\pi}{4}$

(e) $\dfrac{\pi}{2}$

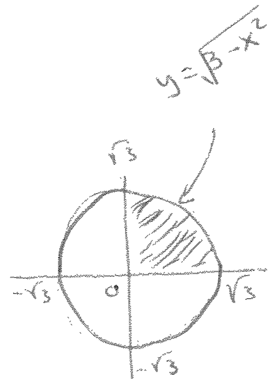
$$= \int_0^{\sqrt{3}} \sqrt{4(3-x^2)}\,dx$$

$$= \int_0^{\sqrt{3}} 2\sqrt{3-x^2}\,dx$$

$$= 2\int_0^{\sqrt{3}} \sqrt{3-x^2}\,dx$$

$$= 2\cdot\frac{1}{4}\pi\left(\sqrt{3}\right)^2$$

$$= \frac{3\pi}{2}$$



7. $\int \frac{\sin(2x)}{\sin x}\,dx =$ $\int \frac{2\sin x\cos x}{\sin x}\,dx = 2\int \cos x\,dx$
$= 2\sin x + C.$

(a) $2\,\sin x + C$

(b) $2\,\cos x + C$

(c) $x + \sin x + C$

(d) $2\,\tan x + C$

(e) $2\,(x - \cos x) + C$

$u = 1 - x \implies du = -dx$

$-\int (2-u)^2 u^5\,du = -\int (4 - 4u + u^2)\,u^5\,du$

$= -\int 4u^5 - 4u^6 + u^7\,du$

$= -\left(\frac{2}{3}u^6 - \frac{4}{7}u^7 + \frac{1}{8}u^8\right) + C$

8. $\int (x+1)^2\,(1-x)^5\,dx =$

$= -\frac{2}{3}(1-x)^6 + \frac{4}{7}(1-x)^7 - \frac{1}{8}(1-x)^8 + C$

(a) $-\frac{1}{8}(1-x)^8 + \frac{4}{7}(1-x)^7 - \frac{2}{3}(1-x)^6 + C$

(b) $\frac{1}{8}(1-x)^8 + \frac{2}{7}(1-x)^7 - \frac{1}{6}(1-x)^6 + C$

(c) $-\frac{3}{4}(1-x)^8 + \frac{2}{7}(1-x)^7 - \frac{1}{3}(1-x)^6 + C$

(d) $4(1-x) - 2\,(1-x)^2 + \frac{1}{3}(1-x)^3 + C$

(e) $-\frac{3}{8}(1-x)^8 - \frac{4}{7}(1-x)^7 + \frac{5}{6}(1-x)^6 + C$

9. The **area of the surface** generated by rotating the curve

$$y = x^2, \ \ 1 \le x \le 2$$

about the y-axis is equal to

$S = \int_1^2 2\pi x \sqrt{1+(y')^2}\, dx$

$= 2\pi \int_1^2 x\sqrt{1+4x^2}\, dx$

$= \frac{2\pi}{8}\cdot\frac{2}{3}\,(1+4x^2)^{3/2}\Big]_1^2$

$= \frac{\pi}{6}\left(17^{3/2} - 5^{3/2}\right)$

$= \frac{\pi}{6}\left(17\sqrt{17} - 5\sqrt{5}\right)$

(a) $\frac{\pi}{6}(17\sqrt{17} - 5\sqrt{5})$

(b) $\pi(17\sqrt{17} - 5\sqrt{5})$

(c) $\frac{\pi}{8}$

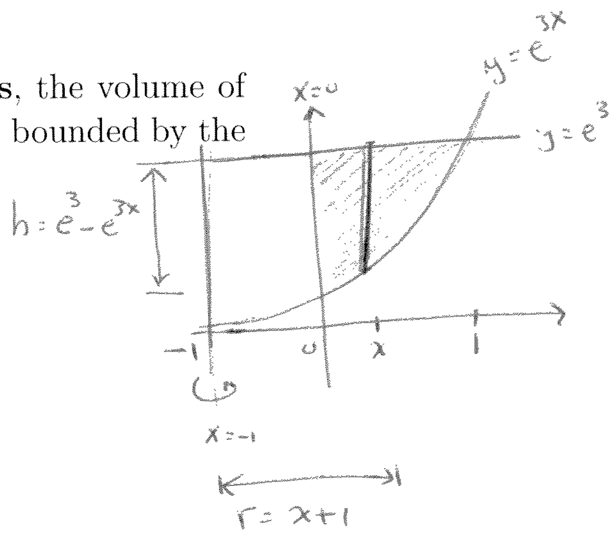
(d) 2π

(e) $\frac{\pi}{12}(17\sqrt{17} - 5\sqrt{5})$

10. Using **the method of cylindrical shells**, the volume of the solid generated by revolving the region bounded by the curves

$$y = e^{3x}, \ \ y = e^3, \ \ x = 0$$

about the line $x = -1$ is given by



$V = \int_0^1 2\pi\,(x+1)\,(e^3 - e^{3x})\, dx$

(a) $\int_0^1 2\pi(x+1)(e^3 - e^{3x})\, dx$

(b) $\int_0^1 2\pi(x-1)(e^3 - e^{3x})\, dx$

(c) $\int_0^e 2\pi(x+1)(e^{3x} - e^3)\, dx$

(d) $\int_1^{e^3} 2\pi y \left(\frac{1}{3}\ln y - 1\right) dy$

(e) $\int_0^1 2\pi\,(x+1)\,(e^{3x} - e^3)\, dx$

11. An equation for the **tangent line** to the curve $y = \int_0^{x^2} t\,e^t\,dt$ at the point $(1, 1)$ is given by

$\frac{dy}{dx} = (x^2 e^{x^2}) \cdot 2x = 2x^3 e^{x^2}$

slope $= \left.\frac{dy}{dx}\right|_{x=1} = 2e$

Equation of the tangent line is

$y - 1 = 2e\,(x-1)$

$y = 2ex - 2e + 1$

(a) $y = 2e\,x + 1 - 2e$

(b) $y = e\,x + 1 - e$

(c) $y = 2e\,x - 1$

(d) $y = -ex + e + 1$

(e) $y = e^2x - e^2 + 1$

12. The **average value** of $f(x) = x^2\sqrt{x+1}$ over $[-1, 0]$ is

$f_{ave} = \frac{1}{0-(-1)} \int_{-1}^{0} x^2\sqrt{x+1}\,dx$

$u = x+1 \Rightarrow du = dx$

$x = -1 \Rightarrow u = 0\,;\ x = 0 \Rightarrow u = 1$

$= \int_0^1 (u-1)^2\sqrt{u}\,du = \int_0^1 (u^2 - 2u + 1)\,u^{1/2}\,du$

$= \int_0^1 (u^{5/2} - 2u^{3/2} + u^{1/2})\,du$

$= \left.\frac{2}{7}u^{7/2} - 2\cdot\frac{2}{5}u^{5/2} + \frac{2}{3}u^{3/2}\right]_0^1$

$= \frac{2}{7} - \frac{4}{5} + \frac{2}{3} - 0$

$= \frac{-18}{35} + \frac{2}{3}$

$= \frac{-54 + 70}{105} = \frac{16}{105}$

(a) $\frac{16}{105}$

(b) $\frac{4}{35}$

(c) $\frac{18}{35}$

(d) $\frac{2}{21}$

(e) $\frac{2}{105}$

13. Suppose f and g are integrable functions on $(-\infty, \infty)$ and that

$$\int_2^5 f(x)\,dx = 4,\ \int_2^{10} f(x)\,dx = 6,\ \int_2^5 g(x)\,dx = -3$$

Which one of the following statements must be **TRUE**?

(a) $\int_2^5 \left(1 - \frac{1}{3}g(x)\right) dx = 4$

(b) $\int_2^5 |g(x)|\,dx = 3$

(c) $\int_2^5 [3\,f(x) - g(x)]\,dx = 9$

(d) $\int_5^{10} f(x)dx = -2$

(e) $\int_2^5 f(x)g(x)\,dx = -12$

$= \int_2^5 1\,dx - \frac{1}{3}\int_2^5 g(x)\,dx$

$= x\Big]_2^5 - \frac{1}{3}\cdot(-3)$

$= (5-2) + 1 = 3+1 = 4$

$u = x^2+3x+1 \Rightarrow du = (2x+3)\,dx$

$x=0 \Rightarrow u=1\ ;\ x=1 \Rightarrow u=5$

14. $\int_0^1 \frac{10x+15}{(x^2+3x+1)^3}\,dx =$

(a) $\frac{12}{5}$

(b) $-\frac{24}{5}$

(c) $\frac{6}{25}$

(d) $\frac{3}{10}$

(e) $\frac{4}{25}$

$\int_0^1 \frac{5(2x+3)}{(x^2+3x+1)^3}\,dx$

$= 5\int_1^5 \frac{1}{u^3}\,du = 5\int_1^5 u^{-3}\,du$

$= 5\cdot \frac{u^{-2}}{-2}\Big]_1^5 = -\frac{5}{2}\cdot\left(\frac{1}{25}-1\right)$

$= -\frac{5}{2}\cdot\frac{-24}{25}$

$= \frac{12}{5}$

15. $\int_{-\pi}^{\pi} (x + \sin x)^5 \, dx =$

(a) 0

(b) 2π

(c) $2\pi^2$

(d) π^6

(e) $2\int_0^{\pi} (x + \sin x)^5 \, dx$

$f(x) = (x + \sin x)^5$

$f(-x) = (-x + \sin(-x))^5$

$= (-x - \sin x)^5$

$= (-1)^5 (x + \sin x)^5$

$= -(x + \sin x)^5 = -f(x)$

So f is an odd function on $[-\pi, \pi]$.

Thus $\int_{-\pi}^{\pi} (x + \sin x)^5 \, dx = 0$

16. The **area** of the region enclosed by the curves $x = y^2$ and $x = y + 2$ is equal to

(a) $\frac{9}{2}$

(b) 3

(c) $\frac{7}{2}$

(d) $\frac{5}{4}$

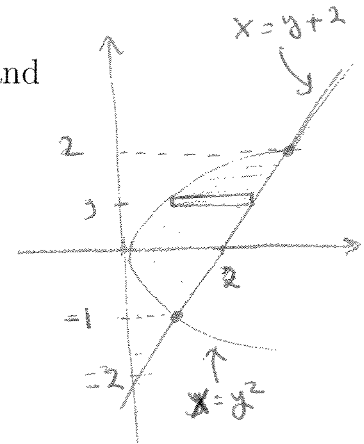
(e) 4

pts of intersection

$y^2 = y + 2 \implies y^2 - y - 2 = 0$

$\implies (y-2)(y+1) = 0$

$\implies y = 2, -1$



$A = \int_{-1}^{2} (y+2) - y^2 \, dy$

$= \int_{-1}^{2} y + 2 - y^2 \, dy$

$= \frac{1}{2}y^2 + 2y - \frac{1}{3}y^3 \Big]_{-1}^{2}$

$= \left(2 + 4 - \frac{8}{3}\right) - \left(\frac{1}{2} - 2 + \frac{1}{3}\right)$

$= 8 - \frac{1}{2} - 3 = 5 - \frac{1}{2} = \frac{9}{2}$

17. If f is an **odd function**, $\int_{-1}^{2} f(t)\,dt = 5$, and $\int_{-1}^{4} f(-t)\,dt = 6$, then $\int_{2}^{4} f(t)\,dt =$

$$\int_{-1}^{4} -f(t)\,dt = 6 \Rightarrow \int_{-1}^{4} f(t)\,dt = -6$$

(a) -11

(b) 1

(c) -1

(d) 11

(e) 0

$-1 \quad 2 \quad 4$

$$\int_{-1}^{4} f(t)\,dt = \int_{-1}^{2} f(t)\,dt + \int_{2}^{4} f(t)\,dt$$

$$-6 = 5 + \int_{2}^{4} f(t)\,dt$$

$$\Rightarrow \int_{2}^{4} f(t)\,dt = -6-5 = -11$$

18. The **length** of the curve

$$y = \int_0^x \frac{e^{2t}-1}{2e^t}\,dt, \qquad 0 \le x \le \ln 2$$

is equal to

- $y' = \frac{e^{2x}-1}{2e^x} = \frac{e^x - e^{-x}}{2}$
- $(y')^2 = \frac{e^{2x}-2+e^{-2x}}{4}$
- $1+(y')^2 = 1 + \frac{e^{2x}-2+e^{-2x}}{4} = \frac{e^{2x}+2+e^{-2x}}{4} = \left(\frac{e^x+e^{-x}}{2}\right)^2$
- $\sqrt{1+(y')^2} = \frac{e^x+e^{-x}}{2}$
- $L = \int_0^{\ln 2} \sqrt{1+(y')^2}\,dx$

$$= \int_0^{\ln 2} \frac{e^x+e^{-x}}{2} = \frac{1}{2}\cdot \left. e^x - e^{-x} \right]_0^{\ln 2}$$

$$= \frac{1}{2}\left[\left(2-\frac{1}{2}\right) - (1-1)\right]$$

$$= \frac{1}{2}\cdot\frac{3}{2} = \frac{3}{4}$$

(a) $\frac{3}{4}$

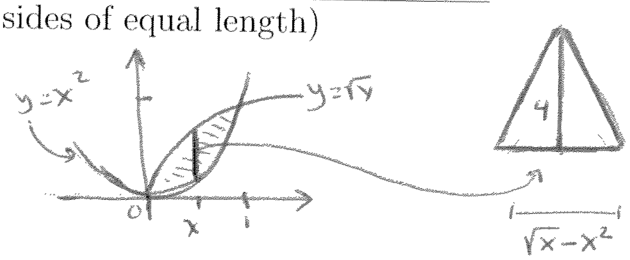
(b) 3

(c) $\frac{1}{2}$

(d) -3

(e) $\frac{3\pi}{4}$

19. The base of a solid is the region bounded by the curves $y = x^2$ and $y = \sqrt{x}$. If the cross sections of the solid perpendicular to the x-axis are **isosceles triangles** of height 4, then the **volume** of the solid is (an isosceles triangle is a triangle that has two sides of equal length)



(a) $\frac{2}{3}$

(b) $\frac{1}{3}$

(c) $\frac{4}{3}$

(d) $\frac{5}{2}$

(e) $\frac{1}{2}$

$A(x) = \frac{1}{2}(\sqrt{x} - x^2) \cdot 4 = 2(\sqrt{x} - x^2)$

$V = \int_0^1 A(x)\,dx$

$= 2\int_0^1 (\sqrt{x} - x^2)\,dx$

$= 2 \cdot \frac{2}{3}x^{3/2} - \frac{1}{3}x^3 \Big]_0^1$

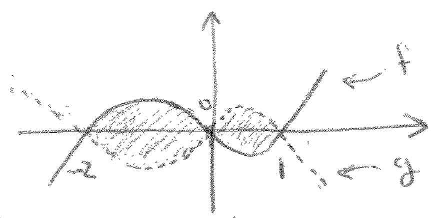
$= 2\left(\frac{2}{3} - \frac{1}{3}\right) = \frac{2}{3}$

20. The **area** of the region bounded by the curves $y = x^3 + x^2 - 2x$ and $y = -(x^3 + x^2 - 2x)$ is equal to

f

g

$y = 0 \Rightarrow x(x^2 + x - 2) = 0 \Rightarrow x(x+2)(x-1) = 0 \Rightarrow x = -2, 0, 1$



(a) $\frac{37}{6}$

(b) $\frac{39}{5}$

(c) $\frac{31}{3}$

(d) $\frac{41}{2}$

(e) $\frac{5}{4}$

$A = \int_{-2}^{0} (f - g)\,dx + \int_0^1 (g - f)\,dx$

$= \int_{-2}^{0} 2(x^3 + x^2 - 2x) + \int_0^1 -2(x^3 + x^2 - 2x)\,dx$

$= 2\left(\frac{1}{4}x^4 + \frac{1}{3}x^3 - x^2\right)\Big]_{-2}^{0} - 2\left(\frac{1}{4}x^3 + \frac{1}{3}x^3 - x^2\right)\Big]_0^1$

$= 2\left(0 - \left(4 - \frac{8}{3} - 4\right)\right) - 2\left(\frac{1}{4} + \frac{1}{3} - 1 - 0\right)$

$= \frac{16}{3} + \frac{5}{6} = \frac{32 + 5}{6} = \frac{37}{6}$