

Instructions: Show Your Work!

1. (4 pts) Find the limit, if exists

(a) $\lim_{x \rightarrow 1} \frac{x^2 - 1}{1 - \sqrt{x}}$

(b) $\lim_{x \rightarrow 1^+} \sqrt{x - 1} \cdot \cos\left(\frac{\pi}{x - 1}\right)$.

2. (3 pts) Use the ϵ, δ definition of a limit to prove that

$$\lim_{x \rightarrow 2} \frac{x - 3}{2} = -\frac{1}{2}.$$

3. (3 pts) Consider the function

$$f(x) = \begin{cases} 8 - x^2 & \text{if } x \leq -4, \\ \lceil 2x \rceil & \text{if } -4 < x < 0, \\ \frac{1}{x^2 - 4} & \text{if } x \geq 0 \end{cases}$$

At each of the points $-5, -4, -\frac{1}{2}$ and 0 , is f continuous?
Justify your answer.

Instructions: Show Your Work!

1. (4 pts) Find the limit, if exists

(a) $\lim_{x \rightarrow 2} \frac{x^2 - 4}{\sqrt{x} - \sqrt{2}}$

(b) $\lim_{x \rightarrow 1^+} \sqrt{x-1} \cdot \cos\left(\frac{\pi}{x-1}\right)$.

2. (3 pts) Use the ϵ, δ definition of a limit to prove that

$$\lim_{x \rightarrow 1} \frac{x-3}{2} = -1.$$

3. (3 pts) Consider the function

$$f(x) = \begin{cases} 8 - x^2 & \text{if } x \leq -4, \\ \lceil 2x \rceil & \text{if } -4 < x < 0, \\ \frac{1}{x^2 - 4} & \text{if } x \geq 0 \end{cases}$$

At each of the points $-5, -4, -\frac{1}{2}$ and 0 , is f continuous?
Justify your answer.