Math 101-151-Class Test III

Name: ID Serial:

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1) If $f(x) = 4 - x^2$, $\quad where \quad -3 \le x \le 1$, find the absolute extreme values.

$f'(x) = -2x = 0 \quad \to \quad x = 0 \;\; and$ 1 pt.

$f'(x)\; DNE\; at\; x = -3 \;\&\; x = 1$

$f(0) = 4 \quad \& \quad f(-3) = -5 \quad \& \quad f(1) = 3$ 1 pt

$f(x)$ has absolute max at $x = 0$, max $= 4$ 1 pt

$f(x)$ has absolute min at $x = -3$, min $= -5$ 1 pt

2) If $f(x) = x^2\sqrt{6-x}$,
   a) Find the intervals on which the function is increasing and decreasing.
   b) Find the values of the local extrema, and indicate if these values are minima or maxima.

1. Domain $f: (-\infty, 6]$ 1 pt
2. To find the critical numbers

$$f'(x) = -\frac{x^2}{2\sqrt{6-x}} + (2x)\sqrt{6-x} = 0$$ 1 pt

$$\frac{x^2}{2\sqrt{6-x}} = (2x)\sqrt{6-x}$$

$$(4x)(6-x) = x^2$$

$$24x - 5x^2 = 0 \quad \to \quad x(24-5x) = 0 \quad \to \quad x = 0 \;\; or \;\; x = \frac{24}{5}$$

The critical values $x = 0 \;\; and \;\; x = \frac{24}{5}$ 2 pts

3. $f(x)$ increasing on $\left(0, \frac{24}{5}\right)$ 1 pt
4. $f(x)$ decreasing on $(-\infty, 0) \cup \left(\frac{24}{5}, 6\right)$ 1 pt
5. $f(x)$ has a local min when $x = 0$ 1 pt
6. $f(x)$ has a local mas when $x = \frac{24}{5}$ 1 pt

3) Let $f(x) = \begin{cases} x^3 & -2 \le x \le 0 \\ x^2 & 0 < x \le 2 \end{cases}$

a) Does f satisfy the hypothesis of the Mean Value Theorem? Explain.

b) If yes, find the value c that satisfies your conclusion.

$f(x)$ continuous over the interval $[-2,2]$ 1 pt

$f(x)$ differentiable over the interval $(-2,2)$ 1 pt

$f'(x) = \begin{cases} 3x^2 & -2 < x \le 0 \\ 2x & 0 < x < 2 \end{cases}$ 1 pt

$f'(c) = \frac{f(2)-f(-2)}{2+2} = \frac{4+8}{4} = 3$ 1 pt

$3c^2 = 3 \quad c^2 = 1 \quad c = \pm 1 \quad c = 1 \quad rejected$ 2 pts

$2c = 3 \quad c = \frac{3}{2}$ 1 pt

The values of $c = -1$ and $c = \frac{3}{2}$

4) A rectangle has its base on the *x-axis* and its upper two vertices on the parabola $y = 12 - x^2$, what is the largest area the rectangle can have, and what are its dimensions?

The area

$A = 2xy = 2x(12 - x^2) = 24x - 2x^3$

1 pt

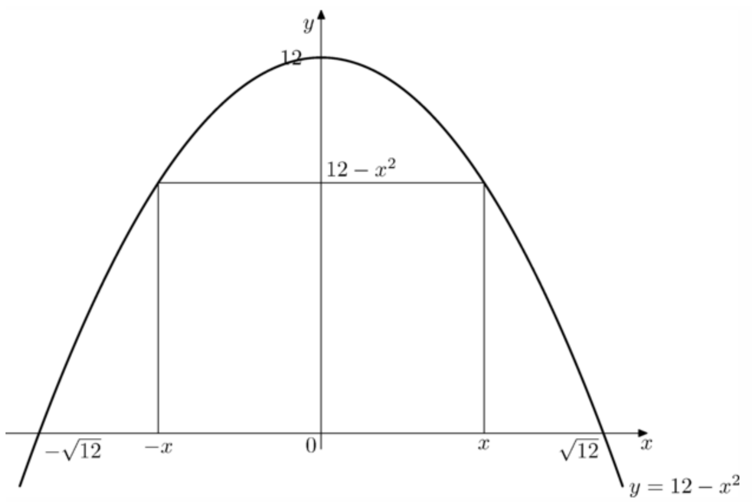
$A' = 24 - 6x^2 = 0$

$6x^2 = 24$

$x^2 = 4 \quad \rightarrow \quad x = \pm 2$ 1 pt

$A'' = -12x$

$A''(-2) = 24 > 0 \quad min$



$A''(2) = -24 < 0 \quad max$

The largest area at $x = 2$

The dimensions $2x = 4$ and $y = 12 - 2^2 = 8$ 1 pt

The largest area $A = xy = 4(8) = 32$ 1 pt

5) Find

a) $\lim_{x\to\infty}\left(\frac{x+2}{x-1}\right)^x$

b) Bonus: Find $\lim_{x\to\infty}\left(\frac{x+2}{x-1}\right)^x$ using a different method than the one used in a) above.

a) Let $y=\left(\frac{x+2}{x-1}\right)^x \quad \rightarrow \ln y = x\ln\left(\frac{x+2}{x-1}\right)$ 1 pt

$$\lim_{x\to\infty}\ln y=\lim_{x\to\infty}x\ln\left(\frac{x+2}{x-1}\right)=\lim_{x\to\infty}\frac{\ln\left(\frac{x+2}{x-1}\right)}{\frac{1}{x}}$$ 1 pt

$$=\lim_{x\to\infty}\frac{\ln(x+2)-\ln(x-1)}{-\frac{1}{x^2}}$$ 1 pt

$$=\lim_{x\to\infty}\frac{\left(\frac{1}{x+2}\right)-\left(\frac{1}{x-1}\right)}{-\frac{1}{x^2}}$$ 1 pt

$$=\lim_{x\to\infty}\frac{3x^2}{(x-1)(x+2)}=\lim_{x\to\infty}\frac{3x^2}{x^2+x-2}=3$$ 1 pt

$$\lim_{x\to\infty}y=\lim_{x\to\infty}e^{x\ln\left(\frac{x+2}{x-1}\right)}=e^3$$ 1 pt

b) $\lim_{x\to\infty}\left(\frac{x+2}{x-1}\right)^x=\lim_{x\to\infty}\left(1+\frac{3}{x-1}\right)^x$ 1 pt

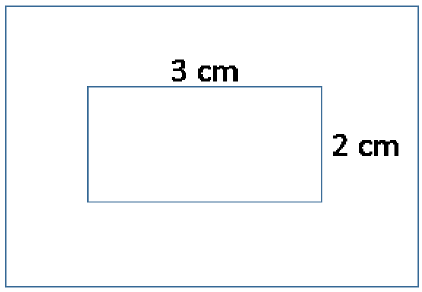
$=\lim_{u\to\infty}\left(1+\frac{3}{u}\right)^{u+1}$ where $u = x-1$ 1 pt

$=\lim_{u\to\infty}\left(1+\frac{3}{u}\right)^u\left(1+\frac{3}{u}\right)=e^3$ 1 pt

6) A rectangle has the following dimensions: 3 cm by 2 cm. The 2 cm side is expanding at the rate of 4 cm/sec and the proportions of the rectangle never change. How fast is the area of the rectangle increasing when its dimensions are 12 cm by 8 cm?

$$x = 12\ cm, \quad y = 8\ cm, \qquad \frac{dy}{dt} = 4\ cm/sec, \qquad \frac{dA}{dt} = ?$$

Since the proportions of the rectangle never change



$2x = 3y$ 1 pt

$2\frac{dx}{dt} = 3\frac{dy}{dt}$ 1 pt

$\frac{dx}{dt} = \frac{3}{2}\ dy = \frac{3}{2}(4) = 6\ cm/sec$ 1 pt

The area

$$A = xy$$

$\frac{dA}{dt} = \frac{dx}{dt}\ y + \frac{dy}{dt}\ x$ 1 pt

$= 6(8) + 4(12)$

$= 96\ cm^2/sec$ 1 pt